MODELLING SHIP’S ROUTING BOUNDED BY THE CYCLE TIME FOR MARINE LINER

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Key words: Ship’s routing, cycle time, branch-and-bound algorithm.

ABSTRACT

This paper addresses the problem of determining an optimal routing that bounded by the cycle time for marine liner. Through exploring the practical planning procedure of shipping company and analyzing the core of route design, this problem is realized as similar as the traveling salesman problem (TSP), but, with some specially industrial properties. A mixed integer programming model is proposed to optimize ship’s routing under satisfying the relationships between the cycle time and deployed vessels with given service frequency in a week. Some constraints, besides, are organized to avoid the routing sequence has separated tours. Intuitively, we also divide the solving procedure into two parts. The first chooses some visited ports from relaxed problem as routing candidates for determining the final routing in the next, both through implemented by the branch-and-bound algorithm. Test results show that our procedure can obtain the suitable route service plan within the stable consumed CPU times of calculation.

INTRODUCTION

Route planning is the source of all operations in marine liner shipping. Carrier has to design the competitive routes for providing public shippers long-termed and dependable shipping conditions for procuring a stable market share. Undoubtedly, demands between port pairs are supposed to be the most important factor for driven the result of route planning. However, they are quite difficult to be estimated the exact amounts for each service trade in the planning stage. The core of planning contents is dedicated to obtain the available ship’s routing and fleet deployment.

1. Problem Description

Route of liner shipping provides a cyclic voyage. The well-defined general traveling salesman problem (TSP) is naturally motivated to compare with it. Some various properties of marine transportation need to be distinguished from TSP and to be reflected in our model formulation, such as:

(1) Although many possible calling ports are considered, not every one has to be visited. Final executed routing sequence is allowed to pass some candidates over.

(2) The number of visited for one port is not limited only once, either. Almost practical two-way routes always call the same port once for both direction services.

Besides, for a special case for the port pairs with relatively larger cargo demands than the vessel’s capacity, a route served between them is designed single-handedly. It implies that the same segment is not served more than twice in most route structures.

Another basic characteristic of marine liner than other transportation modes concerns with the service conditions and deployed vessels. In the normal situation, single vessel can hardly serve long distance route with a fixed service frequency in a week. It needs enough homogeneous vessels, which means same cruising speed at least, to complete the whole rotation under satisfying given weekly frequency. The relationships of them build on the cycle time. We can formulate it as

\[ v = \frac{r}{168} f, \]

where \( v \) means required vessels, \( r \) is the cycle time of voyage in hours, and \( f \) is the given frequency of route service in a week. For example, if we keep one route on weekly service, which means one time visited for each port on the route in a week, seven days trip needs one vessel, fourteen days trip needs two vessels, etc.

Ship’s routing and scheduling problem is a main studied branch of fleet planning and management discussed from Ronen [12]. However, container liner has attracted less attention. Rana and Vickson [10, 11] formulate a nonlinear programming and a mixed integer programming models for multi-vessel and single vessel routing problem, respectively, for containership charter. Cho and Perakis [2] also submit two models to select...
suitable routes for various container fleets in which the fleet size is determined or not. Besides, a series of papers discussed fleet deployment are contributed from Papadakis and Perakis [5], Perakis and Papadakis [7, 8], Perakis and Jaramillo [6], Jaramillo and Perakis [3], Powell and Perakis [9], and Lane et al. [4]. These contributions have not considered the characteristics of liner shipping yet.

2. Practical Planning

In the beginning of the procedure of route planning in marine liner’s practice different available fleets and some possible visiting ports will be picked out after ensuring the service scope of route. Each fleet needs to be assessed one by one for whether it can form a complete, compatible and economic service route. During this assessment, an experimental routing sequence referred to the relationships between the direction of route and the geographical position of ports can be easily designed for inspecting its availability. The cycle time is summed up all its parts from estimation of the sailing time over sea, piloting time in the ports, the staying time in the ports for containers loading and unloading, and allowed buffer time of the type of deployed vessels. If the number of vessels deployed can just make service with the cycle time as held the exact relationships in equation (1), the available ship’s rotation is obtained as well. Otherwise, any revising steps by adjusting the number of visiting ports, routing sequence, or even the type of vessels, cause to return to the top of planning flow again until an available plan obtained. The finished route plan has finally to distribute to relative departments for reconfirming if it can be really executed. This procedure is simply depicted by Fig. 1. From above descriptions, the influence of demands is presented on the type of vessels deployed, selected ports and given service frequency. The crucial contents of route planning are producing a suitable ship’s routing with the limitation of the cycle time.

In this paper, we follow the concept of practice to assume that the capacity allocation caused from the frequency and considered vessels can sustain the demand of cargos for each calling port from early estimation. Besides, it is hard to expect that the cycle time of designed route, which adds all of the fixed sailing and calling times, can exactly match with the number of vessels expressed in equation (1). We apply the concept of buffer time to increase the flexibility of route design. A mathematical model will be formulated to describe this problem in the next section.

MATHMATICAL MODEL

In our model, the segments between port pairs are to be arc variables in considered network for representing the passage of route in the basic model structure. We first define $N$ as the node set of all considered ports as well as $A$ to be the arc set of all considered segments. Each segment is also set buffer time variable to have the flexibility in matching with the cycle time. Some basic constraints will be introduced before the objective function described and a lot of constraints added for avoiding subtours occurred are also explained in following contents.

1. Basic Constraints and The Objective Function

Since the route of liner carrier is always cyclic, arc flows passing each node need to be conserved equally. The variable $x_{ij}$ is defined as the arc variable of the segment between port $i$ to $j$, then flow conservation constraints are represented as equation (2).

$$\sum_j x_{ij} - \sum_i x_{ji} = 0 \quad \forall i \in N$$

The cycle time of route can be actually calculated as representation in equation (1), when carrier has decided the number of deployed vessels and the frequency in a week already. To calculate the cycle time of cyclic journey can easily sum the sailing time of segment and take only the time of staying in one port, origin or destination port, into account to the segment. We further refer a variable of buffer time, $b_{ij}$ with lower and upper bounds ($\beta_{ij}^-$ signed negative and $\beta_{ij}^+$ signed positive respectively), to sailing segment $(i, j)$. Equation (3) describes the constraint of the cycle time, where $t_{ij}$ means the average time of sailing on the segment $(i, j)$ plus staying in port $i$. Equation (4) sets the bounds of buffer time.
\[
\sum_{(i,j) \in A} (t_{ij} + b_{ij}) x_{ij} = \frac{168}{f} v
\quad (3)
\]

\[
b_{ij}^+ \leq b_{ij} \leq \beta^+_{ij} \quad \forall (i, j) \in A
\quad (4)
\]

However, variables \(b_{ij}\) and \(x_{ij}\) make equation (3) as nonlinear that increase the difficulty of problem solved. We replace \(b_{ij}\) to the difference of two nonnegative variables, \(b_{ij}^+\) and \(b_{ij}^-\), i.e. \(b_{ij} = b_{ij}^+ - b_{ij}^-\). Above two equations are replaced by equations (5), (6) and (7) for keeping linear limitations and assuring \(b_{ij}^+\) and \(b_{ij}^-\) always positive between bounds.

\[
\sum_{(i,j) \in A} (t_{ij} x_{ij} + b_{ij}^+ - b_{ij}^-) = \frac{168}{f} v
\quad (5)
\]

\[
b_{ij}^+ - b_{ij}^- \leq \beta^+_{ij} x_{ij} \quad \forall (i, j) \in A
\quad (6)
\]

\[
b_{ij}^- - b_{ij}^+ \leq -\beta^-_{ij} x_{ij} \quad \forall (i, j) \in A
\quad (7)
\]

Some ports are sometimes asked to visit by carrier. We can force the sum of flows directed out from each one of these ports to be the number of compulsory visiting. As representing in equation (8), \(p_i\) means the least number of visiting port \(i\), while \(N_p\) represents the set of all compulsory visiting ports.

\[
\sum_j x_{ij} \geq p_i \quad \forall i \in N_p
\quad (8)
\]

In the variables limitations, \(x_{ij}\) are supposed to be 0-1 integers and buffer time variables \(b_{ij}^+\), \(b_{ij}^-\) are nonnegative real as equation (9).

\[
x_{ij} \in \{0, 1\}, \quad b_{ij}^+, b_{ij}^- \geq 0
\quad (9)
\]

We can formulate the objective function through the above definitions of variables as equation (9) to minimize the sum of variable costs of operation. The parameter of segment cost, \(c_{ij}\), includes the average cost of sailing on the segment \((i, j)\) plus staying in port \(i\) as same reason as the sailing time. Another parameter \(h_{ij}\) represents the unit cost of buffer times on the segment \((i, j)\).

\[
\text{Min. } \sum_{(i,j)} c_{ij} x_{ij} + \sum_{(i,j)} h_{ij} (b_{ij}^+ - b_{ij}^-).
\quad (10)
\]

Above model can decide a route mix with minimization of cost and exact times of sailing on the required segments already, but subtours may occur in the final result. These will also make the route plan impractical. More constraints for avoiding these kinds of results taking place are needed.

2. Constraints for Avoiding Subtour

Normally, another embedded network flow structure is applied to treat the infeasibility from subtours in TSP [1], except the way of exponentially generating a set of constraints for ensuring tour connected. In contrast to TSP the routing problem in this research can allow to bypass ones of all ports. That means the number of nodes connected in the tour is unknown, i.e. the supply or demand on each node has to be changeable in the network flow problem. At the same time, a source of supply node needs to be designated in advance. It is obviously easy to choose a source node from ports in practical. We can assign any one in the set of \(N_p\) or obtain it from the most recommended one by the carrier. This source node is certainly involved in the final result.

We define \(y_{ij}\) as the embedded tour flow on the segment \((i, j)\), \(d_i\) as the supply variable of node \(i\) \((-d_i\) means demand\), and \(d_s\) as the supply of source node specifically. The tour flow is also required to follow the flow conservation passed each node, however supplies or demands on nodes are variables, not given values. We formulate them as equation (11), but it may lose the characteristics of flow conservation due to change the parameters of supply and demand into variables.

\[
\sum_j y_{ij} - \sum_j y_{ji} = \begin{cases} d_i & \text{if } i = s, \\ -d_i & \forall i \in N - \{s\} \end{cases}
\quad (11)
\]

If there are tour flows on the arc, it represents that the arc is selected by the route, and vice versa. The tour flow has a matching relationship with segment flow. We use equation (12) to limit it, where \(M\) means an enough big positive number.

\[
y_{ij} \leq M x_{ij} \quad \forall (i, j) \in A
\quad (12)
\]

The sum of all supplies and demands should be zero, so equation (13) binds the balance of all supplies on nodes. Since every node, except source, is demand node, their sum equals the amount of source node. Meanwhile, equations (13) and (14) satisfy the requirement of assuring the itinerary passed is a whole round trip.

\[
d_s = \sum_{i \in N - \{s\}} d_i
\quad (13)
\]

\[
d_i = \sum_j x_{ij} \quad \forall i \in N - \{s\}
\quad (14)
\]

Finally, equation (15) shows that variables of the tour flow are asked as nonnegative integer and variables of node supply are positive real. Actually, variables of node supply are also supposed to be integer. Equation (14), naturally, satisfies this requirement and equation (13) follows to this condition for source node.

\[
y_{ij} \geq 0 \text{ integer, } d_i \geq 0
\quad (15)
3. Model Formulation

According to the explanation of the evolution for each equation in the previous section, the whole mathematical model [MP] can be pinned up as equation (16-1) to (16-12) where all notations as previous mentions.

\[
\text{[MP]} \quad \text{Min. } \sum_{i,j} (c_{ij} + e_{ij}) x_{ij} + \sum_{i,j} h_{ij}(b^+_{ij} - b^-_{ij}) \quad (16-1)
\]

subject to (16-2), (16-3), (16-4), (16-5), (16-6), and (16-11).

Its solving result can satisfy the service conditions, but subtours may exist. If, on the other hands, all visited ports are connected by the segment flows without subtours, other constraints for avoiding separated tours become redundant naturally. Undoubtedly, the optimal solution is obtained only from the first stage. Otherwise, the visited ports can be the candidates of the next stage’s calculation.

For checking the whole sailing journey is connected or not, an inspecting algorithm is supposed to be developed. However, we instead this complicate procedure to combine into the second phase. First, we sum up the outgoing flows for each node from the result of the first phase optimization. The total flows of outgoing (or incoming) from each node represent the times of node passed by. All of nodes with zero segment flow, that means they are not selected to visit in the first phase, will be disregarded in the next stage’s optimization. Anyone with segment flows can be picked out on purpose as the source of connecting flow, others are considered as demand node. In equation (16-10), values of nodes’ supply are supposed to be the sum of the segment flows, except source. The function of these constraints ensures the visiting tour is connected. It will bind the tour flows in the second phase, if we convey this rule by the result of the first phase. So only to make sure that the node candidates are connected to be same as the one-to-all shortest path problem can increase the flexibility and convenience in optimizing the second phase’s model. Following this concept, values of candidate nodes’ supply can be set up as that let \(d_i\) equals \(1\), in naturally. Some constraints in the second phase model also need to be modified to consist with this idea. In addition to keep the objective function and constraints in the first model, equations (16-7) and (16-9) are revised as

\[
\sum_j y_{ij} - \sum_j y_{ji} = \begin{cases} d_i & \text{if } i = s, \\ -d_i & \forall i \in N - \{s\} \end{cases} \quad (16-7)
\]

\[
y_{ij} \leq M x_{ij} \quad \forall (i, j) \in A \quad (16-8)
\]

\[
d_i = \sum_{i \in N \setminus \{s\}} d_i \quad (16-9)
\]

\[
d_i = \sum_{i \in N \setminus \{s\}} d_i \quad (16-9)
\]

\[
x_{ij} \in \{0, 1\}, \quad b^+_{ij}, b^-_{ij} \geq 0 \quad (16-10)
\]

\[
y_{ij} \geq 0 \text{ integer, } d_i \geq 0 \quad (16-12)
\]

Some side constraints make this model as a mixed integer programming (MIP) problem, although there are two kinds of flows embedded on the defined network. In addition to use the direct calculating methods, such as the branch-and-bound algorithm, to solve it, an intuitive method to manage these two flows respectively is motivated.

**COMPUTATIONAL ALGORITHM**

We propose a two-phase procedure based on the branch-and-bound algorithm to be adopted for the described model. In the first phase, the branch-and-bound algorithm only solves the segment flow problem:

\[
\text{[P1]} \quad \text{Minimizes } (16-1)
\]

subject to (16-2), (16-3), (16-4), (16-5), (16-6), and (16-11).

\[
\sum_j y_{ij} - \sum_j y_{ji} = \begin{cases} k - 1 & \text{if } i = s, \\ -1 & \forall i \in N_f - \{s\} \end{cases} \quad (17)
\]

where \(N_f\) is the set of visited ports from the result of the first phase and \(k\) equals its amount, \(|N_f|\). Meanwhile, equation (16-10) is deleted.

The model [P1] neglects some constraints from original model, so its optimal is supposed to be the lower bound of original problem. The objective value and the branching order of [P1] can be exploited in the second phase. Suppose the objective value of the second phase is equal to the objective value of the first, it is certainly to be the optimum. The solving procedure is shown in Fig. 2.

**NUMERICAL EXAMPLE**

We use one of the trans-Atlantic route provided
from Yang Ming Lines in Taiwan for testing our model. It is illustrated in Fig. 3. The service ports in the Unite State of America include Charleston (CHS), Miami (MIA), New Orleans (NOL) and Houston (HOU), which located in the southeast of U.S.A. or around the shore of Gulf of Mexico respectively. In the European ports, it calls Felixstowe (FXT) in the Great Britain, Le Havre (LEH) in France, Bremerhaven (BRV) in German, Rotterdam (RTM) in the Nerthlands and Antwerp (ANR) in Belgium. Yang Ming Company deploys five ships with 1,962 TEU capacities on this route for weekly service. The cycle time is 35 days. The route calls two times in Charleston and Antwerp, one time for the others and serves 11 segments.

We use this case to test nine situations distinguished with the number of ports visited compulsorily. First, we execute solving the whole model, i.e. from equations (16-1) to (16-12), by the branch-and-bound algorithm directly. Then, two-phase procedure is executed by the same cases. During the tests, we use the optimization package CPLEX 6.0 to implement the calculation of the branch-and-bound algorithm (B&B). Test cases still arrange the same cycle time of 35 days for weekly service, so that needs to deploy 5 vessels from equation (1).

Test results solved by the B&B directly are summarized in the Table 1. Since some cases are not so easy to converge within a countable consumed time, we record final upper bound that also means a best feasible solution before stop calculating and lower bound for each case. Case 1, 2, 3, 7 and 8 all obtained the optimal solution within less CPU times, while case 9 almost obtained optimum in spite of requiring much calculating time. On the other hand, case 4, 5, and 6 spent so much CPU time, but still had a lot of gap between upper and lower bounds.

Same cases are tested by the two-phase branch-and-bound algorithm according to the solution procedure described in last section. Results are summarized in Table 2. All cases obtain optimal solution within less CPU times, while case 9 almost obtained optimum in spite of requiring much calculating time. On the other hand, case 4, 5, and 6 spent so much CPU time, but still had a lot of gap between upper and lower bounds.

Table 1. Test results solved by the branch-and-bound algorithm (B&B)

<table>
<thead>
<tr>
<th>Case</th>
<th>Number of ports compulsory visit</th>
<th>Objective value</th>
<th>The difference of bounds</th>
<th>Consumed time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Upper bound</td>
<td>Lower bound</td>
<td>Value</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>394,670.0</td>
<td>394,670.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>375,520.0</td>
<td>375,520.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>375,520.0</td>
<td>375,520.0</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>366,007.5</td>
<td>361,044.2</td>
<td>4,963.3</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>394,670.0</td>
<td>342,738.1</td>
<td>51,931.9</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>349,037.5</td>
<td>330,929.8</td>
<td>18,107.7</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>318,690.0</td>
<td>318,690.0</td>
<td>0.0</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>318,690.0</td>
<td>318,690.0</td>
<td>0.0</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>318,690.0</td>
<td>318,657.9</td>
<td>32.1</td>
</tr>
</tbody>
</table>

Gap = (upper bound - lower bound) × 100/lower bound.

Fig. 2. The procedure of the two-phase branch-and-bound algorithm.

Fig. 3. Service route of test case.
visited ports need, the less segments and ports passing over require. This phenomenon is also seen from the tendency of the numbers of visited ports and passed segments in final results. For satisfying the constraint of the cycle time, cases 4 to 9 all produce one extra segment besides forming a TSP cycle. Only cases 1 and 2 obtain the divided tours in the first phase results. The routings solved in the two phases for case 1 are illustrated in Fig. 4 and 5. Finally, the total CPU times by the two-phase method are less consumed and more stable in comparison with those of the B&B method.

CONCLUSIONS

The completed route planning of marine liner has to make sure the availability of fleet allocation and routing sequence, besides considering the market requirement and service conditions. We formulate the central issue of route planning problem as an integer programming model in this paper. It can be applied to the planning when the demand of transported cargo is not so emphasized. The results solved from the model can obtain the available routing sequence and the buffer time of each sailing segment with minimum cost. Actually, user can easily conduct a timetable by it, once the time of beginning to serve is provided already. Furthermore, we also develop a two-phase procedure based on the branch-and-bound algorithm for decreasing the CPU consumed time of calculation. The results of comparisons with solving by the branch-and-bound directly also show that this procedure is promising. Undoubtedly, the demand of markets should be taken into the future research account as well as some service conditions for the cargo transport, such as the transit time of goods. Finally, based on those contributions of single route, the more complicate issues of shipping network for marine liner could be continued.

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