MATHEMATICAL MODELING AND PREDICTION METHOD OF CONCRETE CARBONATION AND ITS APPLICATIONS

Ming Te Liang*, Wenjun Qu**, and Chih-Hsin Liang***

ABSTRACT

The carbonation process of concrete is principally a diffusion phenomenon. The penetration rate of carbon dioxide depends mainly on the concrete quality and the exposure condition. Based on both Fick's first and second laws of linear diffusion equations, the three-dimensional equation of conservation of mass is expressed. This equation can be reduced to two- and one- dimensional equations of conservation of mass which can predict the carbonation depth beneath corners and the general surface of concrete structures, respectively. The objectives of this investigation are to measure the depth of carbonation from the free surface of a concrete member and to predict the carbonation depth using a statistical method. The result of the present study indicates that the maximum carbonation depth of concrete at corners is larger than $\sqrt{2}$ times that of the general surface under the conditions of homogeneous, isotropic and uncracked concrete subjected to static load. The carbonation depths of ten existing reinforced concrete (RC) bridges in Taiwan were predicted by the statistical method. The predicted results can be offered as a reference basis of repair, reinforcement or demolition for the existing concrete structures.

INTRODUCTION

The reinforcing steel bars embedded in concrete are protected from corrosion by a thin oxide film that forms on their surface due to the highly alkaline, with pH values above 12.5 (Somush *et al.* 1991; Papadakis *et al.* 1989, 1990, 1991a, 1991b, 1992; Loo *at al.* 1994; Ihekwaba *et al.*1996), environment of the surrounding concrete. This alkalinity occurs calcium hydroxide ($Ca(OH)_2$) produced during the reaction between water and the constituents of cement which occurs the hardening and development of strength of cement and concrete. Carbon dioxide (CO_2) in air penetrates into concrete and reduces the pH value less than 11, that can result in the corrosion of embedded reinforcing steel bars which cause concrete to spall or split. This process is called carbonation that is principally a diffusion phenomenon and the rate of penetration of CO_2 depends mainly on concrete quality and exposure conditions.

Papadakis *et al.* (1991a) mathematically presented and modeled the physicochemical processes in concrete carbonation. The study shows good agreement in the dependence of carbonation depth with time, water-cement ratio, aggregate-cement ratio, atmospheric concentration of CO_2, ambient temperature, and relative humidity (RH). Papadakis *et al.* (1991b) used an accelerated carbonation apparatus to investigate concrete carbonation problems. They also extended the mathematical model developed recently to include the entire range of ambient relative humidities. Both the thermogravimetric analysis and the carbonation depth results show that a carbonation front is indeed formed when the relative humidities is above 50%. When a lower RH values is obtained, no carbonation front is formed. So the kinetics of carbonation reactions become important. Loo *et al.* (1994) reported an experimental study to establish the influence of ambient CO_2 concentration, exposure temperature and curing duration on the rate of carbonation in concrete. They provided many prediction models for the carbonation coefficient by using accelerated carbonation test of concrete. As to the effects of carbonation attack on the corner reinforcing bars of concrete structures, Saeta *et al.* (1995) studied using two-dimenional finite element model. They pointed out that the maximum carbonation depth of concrete structure at corners is larger than 40% that of a general surface in a concrete structure.

The objectives of this paper are to determine the carbonation depth from the surface and at the corners of a concrete member. The carbonation depth is predicted by using a statistical method. In order to investigate the concrete carbonation problem, the three-dimensional equation of carbonation of mass, based on both the Fick

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first and second laws, will be reduced into a one-dimensional diffusion equation of which the solution is simplified as an empirical formula. This empirical formula associated with the statistical method can predict the carbonation depth of the concrete structures.

MATHEMATICAL MODELING OF CONCRETE CARBONATION

Assume that concrete is a kind of homogeneous and isotropic material and is free of crack. This means that the diffusion coefficient, D, of CO_2 in any direction of concrete is all the same.

Applying the concept of mass conservation, the mass flux per unit volume is equal as shown in Fig. 1 and is written in the three-dimensional equation of conservation of mass (Daily and Harleman 1966) as

$$\frac{\partial C}{\partial t} + \frac{\partial \eta_{CO_2,x}}{\partial x} + \frac{\partial \eta_{CO_2,y}}{\partial y} + \frac{\partial \eta_{CO_2,z}}{\partial z} = \gamma_{CO_2} \qquad (1)$$

where C stands for CO_2 mass of unit volume concrete, t for time, $\eta_{CO_2,x}$, $\eta_{CO_2,y}$, and $\eta_{CO_2,z}$ for the mass flux of CO_2 in the x-, y-, and z-direction, respectively, and γ_{CO_2} for absorbed CO_2 mass per unit volume per unit time or the absorbed CO_2 velocity per unit volume. Equation (1) can be expressed in terms of vector form

$$\frac{\partial C}{\partial t} + \nabla \cdot \eta_{CO_2} = \gamma_{CO_2} \qquad (2)$$

where ∇ stands for Laplacian operator.

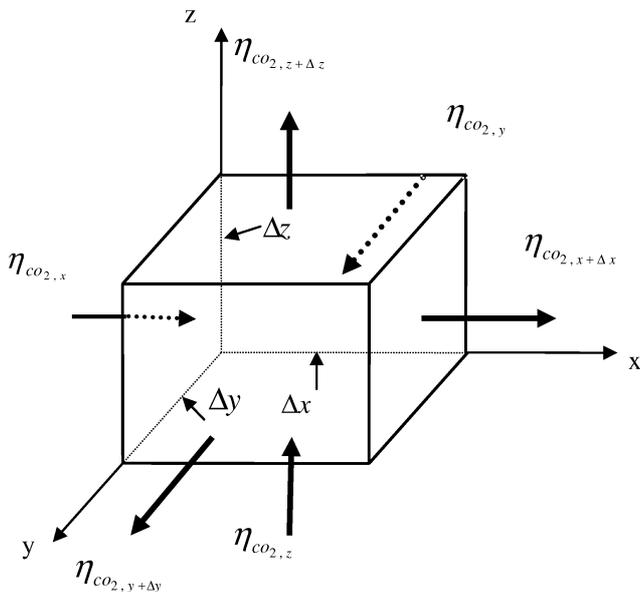Figure 2 denotes CO_2 mass flux in the two-dimensional case. The diffusion of medium CO_2 in concrete can be calculated by Fick's law. Fick's law can be stated as that the rate of diffusion $\frac{dm}{dt}$ (i.e. the mass through surface area S per unit time) is proportional to the production of surface area S and concentration ($\frac{\partial C}{\partial x}$, $\frac{\partial C}{\partial y}$). That is, $\frac{d}{dt} m_{CO_2,x} = -DS_y \frac{\partial C}{\partial x}$ and $\frac{d}{dt} m_{CO_2,y} = -DS_x \frac{\partial C}{\partial y}$. Since $\frac{d}{dt} m_{CO_2,x} = \eta_{CO_2,x} \cdot S_y$ and $\frac{d}{dt} m_{CO_2,y} = \eta_{CO_2,y} \cdot S_x$, we obtain $\eta_{CO_2,x} = -D \frac{\partial C}{\partial x}$ and $\eta_{CO_2,y} = -D \frac{\partial C}{\partial y}$. The reduction of Eq. (2) into two-dimensional case is

$$\frac{\partial C}{\partial t} = D\nabla^2 C + \gamma_{CO_2} \qquad (3)$$

Equation (3) represents the diffusion equation in unsteady state of CO_2 in the corner of concrete structures. Figure 3 shows that the cross-sectional area of concrete exposed in air can be divided into carbonated, carbonation reaction and uncarbonated zones. In the carbonated zone, the reduction of concrete absorbed CO_2 has been finished, i. e., $\gamma_{CO_2} = 0$ in Eq. (3). Thus, Eq. (3) changes to

$$\frac{\partial C}{\partial t} = D\nabla^2 C \qquad (4)$$

Equation (4) is called the Fick second law of linear diffusion equation.

Assume that the ability of concrete absorbed CO_2 in unit volume is m_0 (kg/m³). The CO_2 mass at the carbonation front by concrete with volume $dx_0 \cdot dy_0 \cdot 1$ during dt time interval is
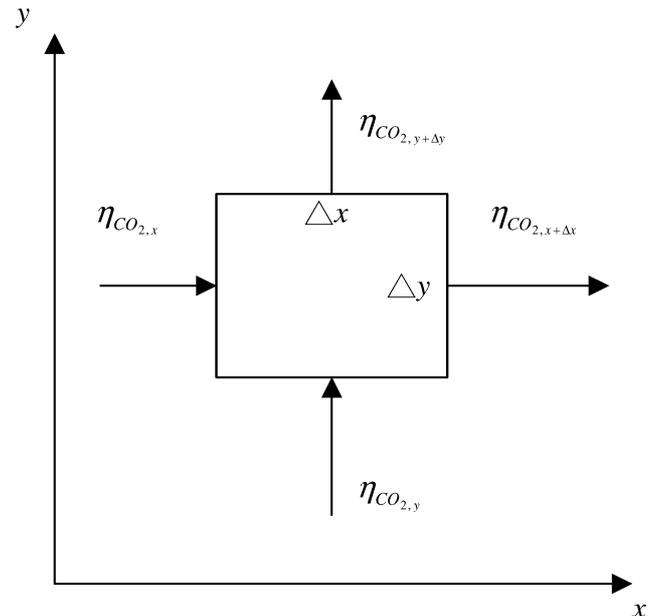

Fig. 1. Mass flux in a three-dimensional control element.

Fig. 2. CO_2 mass flux in the x- and y-direction.

$$dm_{abs.} = m_0 dx_0 dy_0. \tag{5}$$

Assume that CO_2 concentration at infinitesimal time interval is unchanged. Due to CO_2 diffusion, the CO_2 mass of any element is able to reach the carbonation front. This phenomenon can employ to Fick's first law and shown as

$$dm_{diff.} = dm_{CO_{2,x}} dt + dm_{CO_{2,y}} dt$$

$$= -D[dy_0 \left.\frac{\partial C}{\partial x}\right\|_{(x_0, y_0)} + dx_0 \left.\frac{\partial C}{\partial y}\right\|_{(x_0, y_0)}] dt \tag{6}$$

At any time t, the element at the carbonation front in the zone of carbonation reaction should satisfy $dm_{abs.} = dm_{diff.}$, that is,

$$m_0 dx_0 dy_0 = -D \left[dy_0 \left.\frac{\partial C}{\partial x}\right\|_{(x_0, y_0)} + dx_0 \left.\frac{\partial C}{\partial y}\right\|_{(x_0, y_0)} \right] dt$$

$$\tag{7}$$

Both the concentrations of $\left.\frac{\partial C}{\partial x}\right\|_{(x_0, y_0)}$ and $\left.\frac{\partial C}{\partial y}\right\|_{(x_0, y_0)}$ in the carbonation reaction region can not accurately be measured. Generally speaking, if concrete is a kind of homogeneous material, then the function of concentration distribution of CO_2 may be obtained. However, the assumption that concrete is a homogeneous material at any local region is not correct. Hence, both of concentration value and concentration

Carbonation

depth x_0

Fig. 3. Carbonation distinguish of concrete.

gradient are not accurately represented. Moreover, using Eq. (7) to predict the carbonation front curve at any time t is very difficult.

The maximum carbonation depth is the major index for evaluating the concrete condition in a bridge. According to physical phenomena, choosing the point x_{j0} at the carbonated reaction front $x_0 = y_0$ is the maximum carbonation depth. Thus, to find the carbonated reaction front $x_0 = y_0$ at any time, it is important to reduce the complex problem.

Under the general condition, the carbonation depth of concrete structures is not large. We may assume that the concentration C_0 at the concrete surface in air decreases to zero at the front point of carbonation reaction zone. Thus, consider the carbonation front x_{j0} at $x_0 = y_0$ at any time t. We have

$$\left.\frac{\partial C}{\partial x}\right\|_{(x_0 = y_0)} = \left.\frac{\partial C}{\partial y}\right\|_{(x_0 = y_0)} = -\frac{C_0}{x_0} = -\frac{C_0}{y_0} \tag{8}$$

where C_0 stands for CO_2 concentration at the outer edge of cross section of concrete structures. Since $x_0 = y_0$ and $dx_0 = dy_0$, the substitution of Eq. (8) into Eq. (7) yields

$$m_0 dx_0 = 2D \frac{C_0}{x_0} dt$$

$$\text{or } m_0 dy_0 = 2D \frac{C_0}{y_0} dt \tag{9}$$

with initial conditions $x_0 = 0$ and $y_0 = 0$ when $t = 0$. We obtain the solution of Eq. (9)

$$x_{j0} = 2 \sqrt{\frac{DC_0}{m_0} t} = B\sqrt{t} \tag{10}$$

where x_{j0} stands for the carbonation depth of concrete on the line $y = x$ beneath the corner of the concrete structure and $B = 2\sqrt{\frac{DC_0}{m_0}}$ stands for the coefficient of carbonation rate at the corner of the concrete structure. Since the depth of carbonation reaction region approaches zero, Eq. (10) is the maximum carbonation depth at the corner of the concrete structures.

Equation (7) can be reduced into a one-dimensional problem as shown in Fig. 4 and can be written as

$$m_0 dx_0 = -D \left.\frac{\partial C}{\partial x_0}\right|_{x_0} dt \tag{11}$$

The concentration of $\left.\frac{\partial C}{\partial x_0}\right|_{x_0}$ in Eq. (11) can not accube calculated. In addition, the carbonation depth of concrete generally is not large. The assumption in the concrete carbonation analysis for two-dimensional case can be adopted and shown in Fig. 5. This means that the CO_2 concentration in concrete is changed from C_0 on the concrete surface in air and linearly reduced to zero at the point of carbonation reaction front. Thus,

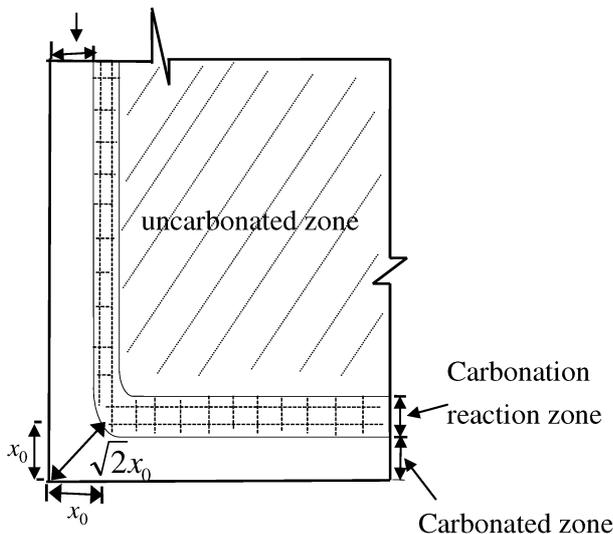consider the depth of carbonation front at time t, and we obtain

$$\frac{\partial C}{\partial x_0}\bigg|_{x_0} = -\frac{C_0}{x_0} \tag{12}$$

Substituting Eq. (12) into Eq. (11) and considering the initial condition, we have

$$\frac{dx_0}{dt} = \frac{DC_0}{m_0}\frac{1}{x_0}, \quad x_0 = 0 \text{ when } t = 0 \tag{13}$$

Therefore, we obtain the solution of Eq. (13)

$$x_0 = \sqrt{\frac{2DC_0}{m_0}t} = A\sqrt{t} \tag{14}$$

where x_0 represents the carbonation depth of concrete under a general surface of concrete structures, and $A = \sqrt{\frac{2DC_0}{m_0}}$ represents the coefficient of carbonation rate under the general surface of concrete structures.

Comparing Eq. (10) with Eq. (14), we obtain

$$B = \sqrt{2}A \tag{15}$$

Equation (15) implies that the maximum carbonation depth of concrete structures at corners is larger than $\sqrt{2}$ times that of a general surface in a concrete structure. The result of Eq. (15) is good agreement with the result obtained by Saetta et al.(1995).

THE PREDICTION OF CONCRETE CARBONATION BY USING A STATISTICAL METHOD

The coefficient of carbonation rate, A, in Eq. (14) is an integrating carbonation response affected by mate-

rial and surrounding medium factors. The higher the value, the less concrete resists carbonation. Because the affected factors of A are almost random variables and influence each other, the best way is first to find the statistical characteristics and then to seek the statistical characteristics of a random function A. However, at the present, there are not enough data to find the A value. Thus, the only directive way to find the A value is by using the method of statistical characteristics of random variable.

In order to find the A value, the sample should be taken for the condition of determined factors. The condition of determined factors are for constant values of water-cement ratio, cement content, admixture, environmental conditions, stress state and wind pressure.

The step of statistical analysis for determining A value is shown as follows. Under the condition of determined factors, we take n numbers of A_i samples and draw the frequency distribution diagram. Based on the frequency distribution diagram, the model of statistical analysis is provided and is verified by either the chi-square test or the Kolmogorov-Smirnov test(Ang and Tang 1975). The value of A_i is determined by the carbonation depth x_i of the ith measurement point, i.e.,
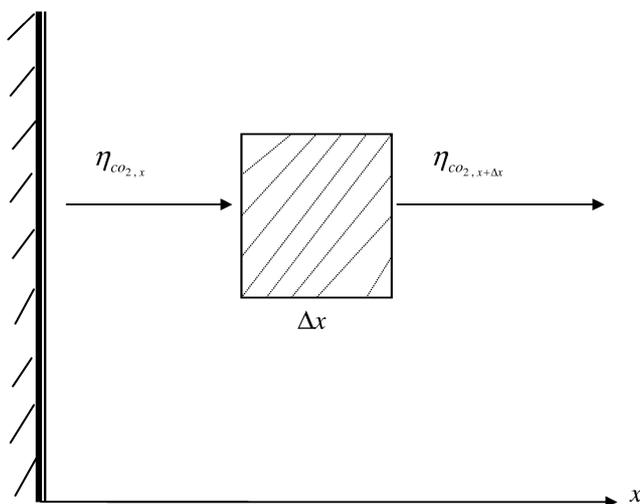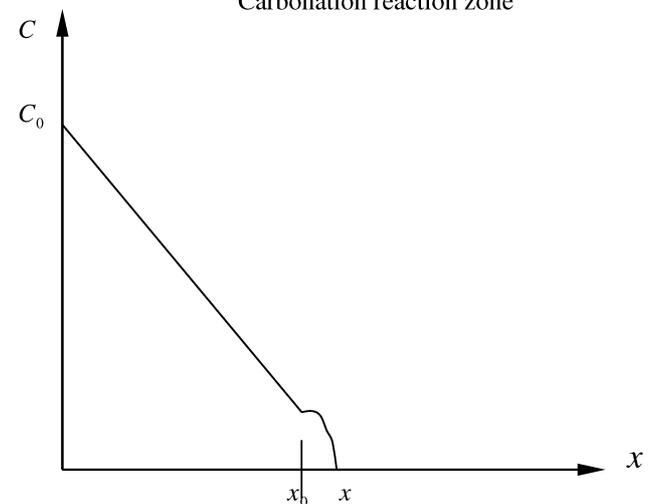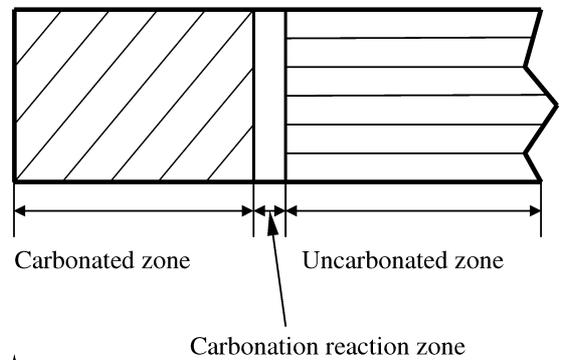

Fig. 5. Carbonation distinguish in the x-direction.

Fig. 4. CO_2 mass flux in the x-direction.

$A_i = x_i/\sqrt{t}$. Clifton(1993) and Liang *et al.*(2000) studied the distribution of carbonation coefficient A and pointed out that A obeyed the normal distribution $N(\mu, \sigma)$, where μ is mean value and σ is standard deviation. Thus, the one-dimensional carbonation depth in a concrete structure is $x_0(t)$, $t \in [0, T]$, where T_s represents a determined time. The normal probability density function at any time t is

$$f(x_0, t) = \frac{1}{\sqrt{2\pi t}\,\sigma_A} \exp\left[-\frac{(x_0 - \mu_A\sqrt{t})^2}{2t\sigma_A^2} \right] \qquad (16)$$

where $\mu_A\sqrt{t}$ is mean value, $\sqrt{t}\,\sigma_A$ is standard deviation, and $f(x_0, t)$ determines the possible result of x_0 at any time t. According to the certain probability P, the constant of β corresponding to P is determined from the table of standard normal probability. The depth of concrete carbonation beneath the general surface after service life t years (Qu 1995)is

$$x_{0\,p\,|t} = (\mu_A + \beta\sigma_A)\sqrt{t} \qquad (17)$$

The geometrical interprelation of Eq.(17) is shown in Fig. 6.

Similarly, based on Eq.(10), we can derive the depth of concrete carbonation at a corner after service life t years

$$x_{0\,p\,|t} = (\mu_A + \beta\sigma_A)\sqrt{2t} \qquad (18)$$

TESTING TECHNIQUES

Cores were extracted from ten different existing RC bridges in Taiwan. For each bridge, numerous cores (see Table 1) of 55 mm diameter with 110 mm height were taken. These specimens were tested for density, compressive strength, and depth of carbonation.

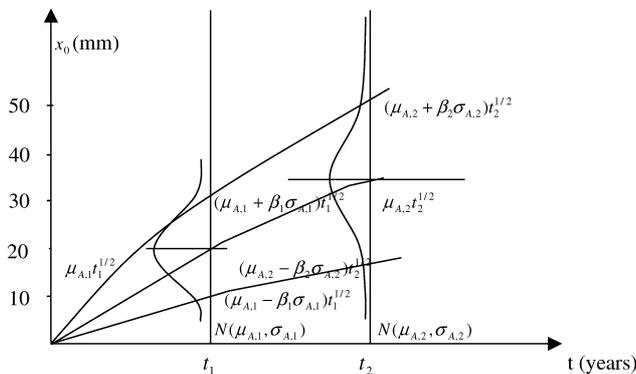Cores were taken from the exposed RC members of bridges. The majority of the bridges considered were reinforced concrete bridges. Cores were extracted and tested according to ASTM-C-39-86(1997). All cores were extracted horizontally from girder, diaphram, capbeam, parapet, deck, pier, abutment, and ramps. The cores were cut using a rotary cutting steel with diamond bits.

The density and compressive strength were not considered in this study. The depth of carbonation was assessed by splitting the concrete cores and spraying the split specimens uniformly with phenolphthalein indicator (a solution of 1% phenolphthalein ($C_{20}H_{14}O_4$) in 70% ethyl alcohol (C_2H_5OH))(Dhir *et al.* 1989). The depth of carbonation was measured with a steel ruler to the nearest 1mm. Several measurements were taken on a given specimen. The average cover thicknesses of each member of the ten bridges were also obtained and are shown in Table 1.

APPLICATIONS

In order to illustrate the application of Eq. (17), we shall use it to predict the carbonation depths of ten RC bridges in Taiwan after service life 50 years. The mean values and standard deviations of carbonation depths measured from ten RC bridges in Taiwan are shown in Table 1. Figure 7 and Fig. 8 denote the frequency distribution and normal distribution $N(\mu_A, \sigma_A)$ of the coefficient of carbonation rate of the Gaang-shi viaduct in Taiwan, respectively. The other bridges' frequency distribution and normal distribution can be obtained as the same way as the Gaang-shi viaduct and are shown in Table 1. Assume that the concrete carbonation depth of the Gaang-shi viaduct has probability 95% (i.e., $p = 0.95$, $\beta = 1.645$ obtained from the table of standard normal probability) (see Fig. 6) after using 50 years in service. Using Eq. (17) and the normal distribution $N(\mu_A, \sigma_A) = (4.180, 2.000)$ (see Fig. 8) of the Gaang-shi
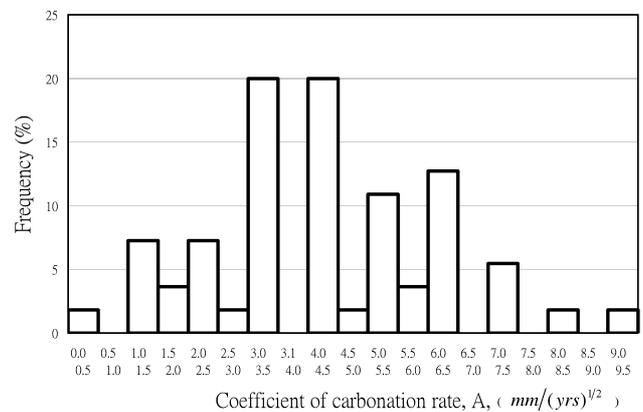
Fig. 6. Probability relationship between concrete carbonation depth and service life.

Fig.7. Frequency distribution of the coefficient of carbonation rate (the Gaang-shi viaduct in Taiwan).

Table 1. Statistical results of carbonation testing and carbonation depths (mm) predicted by statistical method after service life $t = 50$ years for reinforced concrete bridges in Taiwan

Bridge name*	$f_c^{'+}$ (MPa)	w/c#	Service life (yrs)	Test points	Carbonation depth x_0 (mm)			Coefficient of carbonation rate A (mm/(yrs)$^{1/2}$)			Statistical Method $x_{0\,P\,|t} = (\mu_A + \beta\sigma_A)\sqrt{t}$ (mm)
					Mean Value μ_{x0}	Standard Deviation σ_{x0}	Variance	Mean value μ_A	Standard deviation σ_A	Variance	
A	40.35	0.5	49	6	12.77	7.122	0.5579	1.824	1.017	0.5579	24.73
B	48.60	0.5	20	20	19.30	23.060	1.1950	4.314	5.157	1.1950	90.49
C	27.32	0.45	26	6	7.55	8.155	1.0800	1.481	1.599	1.0800	29.07
D	43.01	0.5	18	12	12.79	9.989	0.7809	3.015	2.355	0.7809	48.71
E	26.43	0.45	26	8	19.31	17.980	0.9310	3.788	3.526	0.9310	67.80
F	28.91	0.6	15	18	13.51	9.671	0.7158	3.489	2.497	0.7158	53.72
G	39.85	0.4	24	20	10.37	9.323	0.8995	2.116	1.903	0.8995	37.10
H	27.59	0.55	14	10	10.70	10.760	1.006	2.860	2.876	1.0060	53.68
I	21.93	0.5	24	20	14.77	11.250	0.7620	2.966	2.340	0.7620	48.19
J	20.61	0.5	23	111	20.05	9.580	0.4800	4.180	2.000	0.4800	27.44

*: A: Huey-tong bridge, B: Beei-men viaduct, C: Way- shuang rivulet bridge, D: Her-pyng west road viaduct, E: Jzyh-chyang bridge, F: Ay-gwo west road viaduct, G: old Hwan-nan viaduct, H: Daw-nan bridge, I: Shi-yuan bridge, and J: Gaang-shi viaduct, +: $f_c^{'}$ - the compressive strength of concrete, #: w/c-water-cement ratio
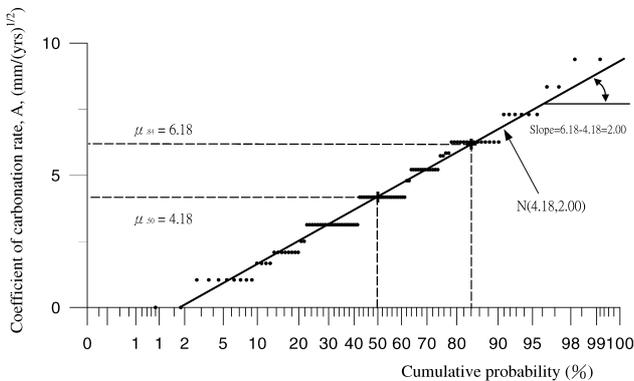

Fig.8. Coefficient of carbonation rate plotted on normal probability paper (the Gaang-shi viaduct in Taiwan).

viaduct, we obtain the carbonation depth

$$x_{0\,0.95}|_{50} = (4.180 + (1.645)\times(2.000))\sqrt{50} = 27.44\ (mm)$$

This means that the carbonation depth of the Gaang-shi viaduct under the general concrete surface is 27.44 mm after service life 50 years as indicated in Table 1. The carbonation depths of the other bridges can similarly obtained and also shown in Table 1. If substitute $t = 50$ years and $A = 4.18$ mm/$(year)^{\frac{1}{2}}$ (see Table 1) into Eq.(14), then the carbonation depth of Gaang-shi viaduct is $x_0 = 29.56$ mm. The difference with values of 7.7% between the predicted values used

Eqs.(14) and (17) is very small and is allowable. This is occurred that the Gaang-shi viaduct has 111 test points. However, in the same manner, if substitute $t = 50$ years and $A = 1.824$ mm $(year)^{\frac{1}{2}}$ (see Table 1) into Eq. (14), then the carbonation depth of Huey-tong bridge is $x_0 = 12.90$ mm. Compared this value with $x_0 = 24.73$ mm (see Table 1) obtained from the statistical method, the difference with values of 46.1% is obviously very large and is not acceptable. This is resulted from that the Huey-tong bridge only has 6 test points.

DISCUSSION

The penetration of carbonation front is a result of a diffusion process with chemical reaction. The content of carbon dioxide can be calculated by using a diffusion equation. Consider the one-dimensional diffusion equation(Crank 1975):

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} - kC \tag{19}$$

with initial and boundary conditions

$$C(x, 0) = C_i \tag{20}$$

$$\text{and } C(x, 0) = C_s \tag{21}$$

where D stands for the diffusion coefficient, k for constant, $C(x, t)$ for the CO_2 content at depth x at time t, C_i for the initial CO_2 content of the concrete, and C_s for

the CO_2 content of the exposed concrete surface. The first and second term of the right-hand side of Eq. (19) represent the diffusion and chemical reaction effect, respectively.

Substituting $C = \overline{C} e^{-kt}$, where $\overline{C}$ is a new variable of CO_2 content(Carslaw and Jaeger 1959), and applying the Laplace transformation to Eqs. (19), (20) and (21), the particular solution is

$$C(x, t) = [C_i + (C_s e^{kt} - C_i) \operatorname{erfc}(\frac{x}{\sqrt{4tD}})] e^{-kt} \qquad (22)$$

where erfc represents the complementary error function. At a given time, the CO_2 content at the depth of the carbonation $x = x_0$ is equal to the value $C = C_0$. In this situation, Eq. (22) can be written as

$$x_0 = \sqrt{4D} \operatorname{erfc}^{-1} (\frac{C_0 e^{kt} - C_i}{C_s e^{kt} - C_i}) \sqrt{t} \qquad (23)$$

The carbonation coefficient, A, may be able to express as

$$A = \sqrt{4D} \operatorname{erfc}^{-1} (\frac{C_0 e^{kt} - C_i}{C_s e^{kt} - C_i}) \qquad (24)$$

Equation (23) may be represented by

$$x_0 = A\sqrt{t} \qquad (25)$$

It is very obvious that Eq. (25) is identical to Eq. (14).

Papadakis *et al.* (1991a) developed a general mathematical model of all physicochemical processes involved in concrete carbonation. This mathematical model can be reduced into

$$x_0 \frac{dx_0}{dt} = \frac{D_{e, CO_2}^c [CO_2]^0}{[Ca(OH)_2]^0 + 3[CSH]^0 + 3[C_3S]^0 + 2[C_2S]^0} \qquad (26)$$

where D_{e, CO_2}^c denotes that the effective diffusivity of CO_2 is constant, $[CO_2]^0$ represents the concentration of CO_2 at surface $x = 0$, $[Ca(OH)_2]^0$, $[CSH]^0$, $[C_3S]^0$ and $[C_2S]^0$ stands for the concentration of $Ca(OH)_2$, $3CaO \cdot 2SiO_2 \cdot 3H_2O$, $3CaO \cdot SiO_2$ and $2CaO \cdot SiO_2$ at $t = 0$, respectively.

Equation (26) can be integrated under the initial condition $x_0 = 0$ at $t = 0$, so Eq. (27) is yielded

$$x_0 = \sqrt{\frac{2D_{e, CO_2}^c [CO_2]^0}{[Ca(OH)_2]^2 + 3[CSH]^0 + 3[C_3S]^0 + 2[C_2S]^0} t} \qquad (27)$$

Equation (27) can be written as the empirical expression of the form $x_0 = A\sqrt{t}$, which is also identical to Eq. (14).

Generally speaking, in order to apply the probability method to treat corrosion data, as many specimens as possible must be taken. Thus, first, it also needs to confirm that the obtained data belongs to the specimen of overall or not from the given knowledge of corrosion system, the probability distribution suited for specimen can be found. In this situation, no matter what specimens are very few, the statistical method can still be analytic. The important problem is that the estimated result is reliability or not. Of course, the more specimens, the more reliability. In other respects, under the uncertain condition of probability distribution, the quantitative conclusion with respect to specimens can not be made. For determining data suited which kind of probability distribution, the suitable testing should be adopted. Under this situation, many specimens ranged from 100 to 200 have to used for statistical analysis. However, if we only depend on the method of statistics for predicting the service life of reinforced concrete structures due to corrosion damage such as carbon dioxide ingress, then we feel that it is not enough. In essence, choosing the suitable probability distribution should be based on the developed mechanism and process of corrosion phenomena.

The influence factor of concrete carbonation can be divided into both material and environmental factors. The material factors include water-cement ratio, cement composition, cement content, admixture, construction quality, curing condition and concrete strength. The environmental factors consist of relative humidity, temperature, external wind pressure, stress state, the concentration of acid medium and exposition time. In order to ensure the predicted carbonation depth obtained from the statistical method, the predicted values should be modified by the concrete material coefficient ζ_1, the cover thickness coefficient of steel ζ_2, a durability coefficient influenced by environment ζ_3, and the structural damage coefficient ζ_4 (Liang *et al.* 1999). Thus, in order to increase the practical use, Eq.(17) may be modified as

$$x_{0\,p|t} = \zeta_1 \zeta_2 \zeta_3 \zeta_4 (\mu_A + \beta \sigma_A) \sqrt{t} \qquad (28)$$

CONCLUSIONS

The carbonation depth is an important index to estimate both the damage and durability of reinforced concrete structures. This paper has presented the mathematical modeling and prediction method of the carbonation depth for the homogeneous, isotropic and uncracked concrete structures subjected to static load. It should be noted that the present study provides the maximum carbonation depth of concrete structures at corners which is larger than $\sqrt{2}$ times that of the general

surface of concrete structures. In addition, the carbonation depths of ten RC bridges in Taiwan were predicted by a statistical method. The investigation shows that the predicted results obtained from the statistical method maybe provide the engineering reference basis of repair, reinforcement or demolition for the existing concrete structures.

ACKNOWLEDGMENTS

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