MODEL CONSTRUCTION OF OPTION PRICING BASED ON FUZZY THEORY

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ABSTRACT

Option pricing is a tool that investors often use for the purpose of arbitrage or hedging. However, both the Black-Scholes model and the CRR model can only provide a theoretical reference value. The volatility in the CRR model cannot always appear in the precise sense because the financial markets fluctuate from time to time. Hence, the fuzzy volatility is naturally to be considered. The main purpose of this paper is the application of fuzzy sets theory to the CRR model. It is expected that fuzzy volatility, instead of the crisp values conventionally used in the CRR model, can provide reasonable ranges and corresponding memberships for option prices, as a result of which, investors can interpret optimal value differently for different risk preferences. This paper shows a new method for option valuation using fuzzy set theory that is based on findings from earlier option valuation methods and from fuzzy membership function. In conclusion, the empirical evidence indicates the effectiveness of the proposed fuzzy model.

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I. INTRODUCTION

Warrants supply investors with choices for financial leverage. When the price of the underlying asset rises, the owner of the warrant can buy the stocks at the specified price, the return will be a simple multiple of the purchased stocks. When the price of the underlying asset goes down, at most the premium is lost. If we can accurately predict the optimal range of an option price, investors can make a profit and hedge against losses from the derivatives.

The Black-Scholes model [3] or binomial tree option pricing model has been widely applied for computing the optimal warrant price. Volatility is assumed to be constant. However, as pointed out by Lauterbach and Schultz [51] and Hauser and Lauterbach [33], the volatility is the most controversial variable. Hence, many subsequent studies have focused on estimating the volatility, for example using historical data or the Parkinson method [66]. The different estimated volatilities certainly result in price variation. In this study, fuzzy set theory is applied to model the volatilities. It is expected that this method will replace the complex models used in the previous studies [6-28, 34-40, 47-50, 52-64, 68-76, 79-84, 88-92, 95-105].

Most recent studies of option pricing have focused on how to relax the assumptions made in the Black-Scholes model and CRR model [31]. These assumptions include: (1) the price fluctuation of the underlying asset must follow a log-normal distribution; (2) the short-term risk-free rate of interest is constant; and (3) the volatility of a stock remains constant. After relaxing these assumptions, we can set up new definitions. For example, we assumed that the price of an underlying asset follows the Poisson Jump-Diffusion Process or the Markov Process. The interest and volatility can also be random processes. In line with the focus in this paper on the volatility, we first discuss some literature regarding option pricing which is relevant to volatility. Fuzzy set theory as well as the binomial tree option pricing model are reviewed in the following section. In Section 3 fuzzy set theory is applied to the binomial tree option pricing model. Some empirical results from the fuzzy model are presented in Section 4. Conclusions to sum up the article are offered in Section 5.

II. LITERATURE REVIEW

Hull and White [46] relaxed the assumption that the distribution of the price of underlying assets and volatility were constant. Wiggins [86], and Scott [67] let go of the assumption that the volatility was constant instead, assumed Stochastic-Volatility. Cox [30] introduced the concept of Constant-Elasticity-of-Variance for volatility. Amin [1] and Scott [67] considered the Jump-Diffusion processes of both stock prices and the volatility to be a random process. These models all endeavored to provide more accurate prices.

1. Fuzzy Set Theory Revisited

The origins of fuzzy sets theory track back to an article by Lotfi Zadeh [94] who stated that an element either belongs to a set or does not belong to a set at all in classical set theory. This type of true or false logic is commonly applied in financial applications. But, bi-value logic was not sufficiently comprehensive to deal with real world problems and presents a problem, because financial decisions are generally made under uncertainty. Consequently, an algebra called fuzzy sets was developed to deal with imprecise elements in our decision making processes, and is the formal body of theory that allows the treatment of practically all decisions in an uncertain environment [29]. Instead, it is more proper to represent items by a degree of membership indicating the degree of belongingness. Since then, this theory has been successfully applied in many problem domains, such as engineering, transportation, management and business.

Zadeh [94] extended the characteristic function to introduce the concept of fuzzy subset which is defined by its membership function that is viewed as an extension of characteristic function [87]. The fuzzy set concept deals with real observations through possibility. The membership function of a fuzzy set is introduced as follows. In traditional relations, we use 0 or 1 to represent the relationship. In fuzzy relations, we instead use a number between 0.0 and 1.0 to show the degree of relationship, as shown in Fig. 1. When the relationship is closer to 1, the relationship is stronger. On the other hand, when the relationship is closer to 0, the relationship is weaker. In fuzzy set theory, the relationship is described through a membership function.

Two basis operators are applied to describe the relationships: an algebraic product and a minimum. The algebraic product is used to describe the propagation of the degree of membership (μ) from one time period to another. Let R and S be two fuzzy sets. Then

$$\mu_{R\cdot S} = \mu_R(x)\cdot\mu_S(y) \tag{1}$$

The minimum is used to calculate the intersection of the degree of membership (μ) of two different fuzzy sets; the maximum is used to represent the union

$$\mu_{R\cap S} = \min(\mu_R(x), \mu_S(y)) \tag{2}$$

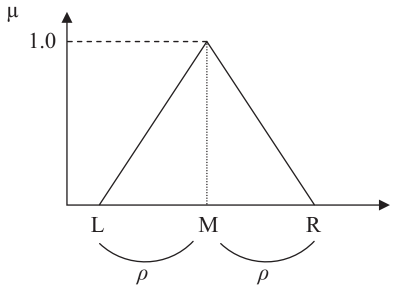

Fig. 1. A triangular fuzzy set.

$$\mu_{R\cup S} = \max(\mu_R(x), \mu_S(y)) \tag{3}$$

2. Option Pricing Model

Options play more and more important roles in the financial market as a widely applied in financial derivatives [62]. Generally, options are commonly valued with three methods. Black-Scholes option pricing formula [3] is also called Black-Scholes model. The binomial option valuation method [31] and Monte-Carlo based methods [4, 29] are also utilized in the present work. Black and Scholes [3] made an important breakthrough by deriving a differential equation that the price of any derivative security dependent on a non-dividend paying stock must satisfy it. Concerning risk-neutral investors, the Black-Scholes pricing formula for a call option is [5]

$$C_0 = S_0 N(d_1) - Xe^{-rT}N(d_2)$$

where

$$d_1 = \frac{\ln(S_0/X) + (r+\sigma^2/2)T}{\sigma\sqrt{T}},\ d_2 = d_1 - \sigma\sqrt{T}$$

C_0 is the price of option; S_0 is the stock price; $N(d)$ is the probability that a random draw from a standard normal distribution which will be less than d; r is the annualized continuously compounded rate with the same maturity as the expiration of the option; X is the exercise price; T is the time of maturity and δ denotes the standard deviation of the annualized continuously compounded rate of return of the stock. Merton [65] extended the Black-Scholes model to dividends-paying stocks as

$$C_0 = S_0 e^{-\delta T}N(d_1) - Xe^{-rT}N(d_2)$$

where

$$d_1 = \frac{\ln(S_0/X) + (r-\delta+\sigma^2/2)T}{\sigma\sqrt{T}},\ d_2 = d_1 - \sigma\sqrt{T}$$

δ denotes the dividends paid out during the life-time of the

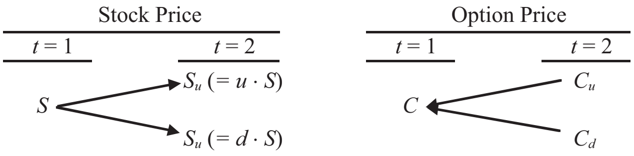

Fig. 2. Stock price movement in time Δt under the binomial model.

option. The standard Black-Scholes model prices a European option on an asset follow a geometric Brownian motion. By using the modification of Black-Scholes formula [87], many methodologies for the option pricing have been proposed. Black-Merton-Schole developed a more general approach that derives a partial differential equation. Wang *et al*. [85] applied partial differential equation in pricing barrier options. Harrison and Kreps [32] developed the martingale approach to arbitrage thory that is the most general method for pricing of contingent claims [2].

3. Binomial Tree Option Pricing Model

Binomial options pricing model originated from Cox *et al*. [31], henceforth CRR model that has simple structure is widely applied in the financial market and is one of the basic options pricing methods [62]. First, the binomial tree option pricing model is introduced with a one-step example ($n = 2$, where n is the total period of time), including an option pricing model and its inference process. Other cases can be derived similarly as n gets larger. Suppose the stock price at period $t = 1$, S, is known. The one-step option pricing model can inference two possible stock prices (up and down movements) at some period $t = 2$, as in Fig. 2. The corresponding call prices are calculated from the stock prices at $t = 2$. Then, we can calculate back to the call price at $t = 1$.

Let P be the probability for S to move up, and

$$P = \frac{a - d}{u - d} \tag{4}$$

where $a = e^{r \cdot \Delta t}$, the factor of discount; r is the risk-free rate; Δt is the length of one time period. Let

$$u = e^{\sigma \cdot \sqrt{\Delta t}} \tag{5}$$

represent the stock price after moving up, where σ is the volatility of stock price. Let $d = 1/u$. Hence,

$$d = e^{-\sigma \cdot \sqrt{\Delta t}} \tag{6}$$

represents the stock price after moving down. Suppose $u > a > d$.

From equation (4), we can calculate

$$1 - P = \frac{u - a}{u - d} \tag{7}$$

As in Fig. 2, we define the following:

$$S_u = u \cdot S \tag{8}$$

$$S_d = d \cdot S \tag{9}$$

where S_u is the stock price for the next period (at $t = 2$) when the stock price moves up; S_d is the stock price for the next period (at $t = 2$) when the stock price moves down.

Meanwhile, from the concept of the call price, let

$$C_u = \max\,(0, S_u - K) \tag{10}$$

$$C_d = \max\,(0, S_d - K) \tag{11}$$

where C_u is the call price at $t = 2$, calculated from S_u; and C_d is the call price at $t = 2$, calculated from S_d; K is the exercised price.

Accordingly, we can get the following:

$$C = \frac{(a - d)C_u + (u - a)C_d}{(u - d)a} \tag{12}$$

Finally, substituting Eqs. (4) and (7) into Eq. (12), we obtain the call price at $t = 1$, which is our goal

$$C = \frac{P \cdot C_u + (1 - P) \cdot C_d}{a} \tag{13}$$

Similarly, we can calculate different stock prices for different total time periods n, and then calculate back to the call price at $t = 1$.

III. FUZZY BINOMIAL TREE OPTION PRICING MODEL

Before set up the model, the following assumptions are allowed: (1) There is no transaction costs, no taxes, no restrictions on short sales, no arbitrage opportunities in the markets and assets are infinitely divisible. (2) The underlying asset does not pay dividends during the life of derivative. (3) The riskless rate of interest is constant and all maturities are the same. After combining binomial tree option pricing model and fuzzy theory, the binomial tree option pricing model is extended to fit a fuzzy binomial tree option pricing model. The results of inferences for fuzzy stock prices and fuzzy call prices are introduced in detailed by Yu *et al*. [93].

1. The Process of Inference for Fuzzy Stock Prices

Similar to the binomial tree option pricing model, we suppose that S is known at $t = 1$. After the fuzzification of σ and $-\sigma$, the up and down movements u and d are replaced by uu, um, and ud, and du, dm, and dd, respectively. uu, um, and ud are three possibilities for the up movement under the largest,

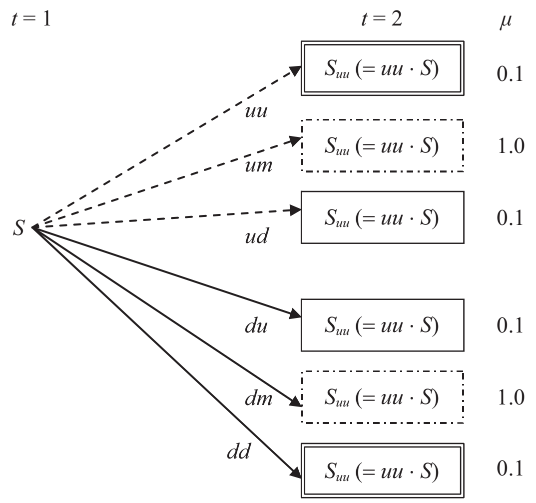

Fig. 3. Stock prices at t = 2.

medium, and smallest volatility, respectively. *uu*, *um*, and *ud* can be defined as follows:

$$uu = e^{(1+\rho)\cdot\sigma\cdot\sqrt{\Delta t}},\ um = e^{\sigma\cdot\sqrt{\Delta t}},\ ud = e^{(1-\rho)\cdot\sigma\cdot\sqrt{\Delta t}}$$

We also let $uu = \frac{1}{dd}$, $um = \frac{1}{dm}$, $ud = \frac{1}{du}$. Hence, $du = e^{-(1-\rho)\cdot\sigma\cdot\sqrt{\Delta t}}$, $dm = e^{-\sigma\cdot\sqrt{\Delta t}}$, $dd = e^{-(1+\rho)\cdot\sigma\cdot\sqrt{\Delta t}}$.

Now, we can list all the possible stock prices through various combinations of *uu*, *um* and *ud*, and *du*, *dm* and *dd*.

All the stock prices at $t = n$ can be derived as follows:

$$uu^a \cdot um^b \cdot ud^c \cdot du^x \cdot dm^y \cdot dd^z \cdot S$$

where a, b, c, x, y, z are integers and can be any combinations under the condition $a + b + c + x + y + z = n - 1$; and n is the total number of time periods.

Suppose there are n time periods. The number of stock prices at $t = n$ are 6^{n-1}.

For example, suppose for t = 2 there are in total 6 stock prices. $a + b + c + x + y + z = n - 1 = 1$. The stock prices at t = 2 are derived as follows: when $a = 1$, $b = c = x = y = z = 0$, $S_{uu} = uu \cdot S$ with $\mu = 0.1$. When $b = 1$, $a = c = x = y = z = 0$, $S_{um} = um \cdot S$ with $\mu = 1.0$. When $c = 1$, $a = b = x = y = z = 0$, $S_{ud} = ud \cdot S$ with $\mu = 0.1$. When $x = 1$, $a = b = c = y = z = 0$, $S_{du} = du \cdot S$ with $\mu = 0.1$. When $y = 1$, $a = b = c = x = z = 0$, $S_{dm} = dm \cdot S$ with $\mu = 1.0$. When $z = 1$, $a = b = c = x = y = 0$, $S_{dd} = dd \cdot S$ with $\mu = 0.1$.

We have the above six equations for stock pricing: see Fig. 3. In this figure, the dash lines with arrows indicate upward movement from t = 1 to t = 2; while the solid lines indicate downward movement. The values in solid boxes (S_{ud} and S_{du}) indicate the stock prices with the smallest volatilities at t = 2; those in dashed boxes (S_{um} and S_{dm}) indicate the stock prices with medium volatilities; and those in double boxes (S_{uu} and S_{dd}) indicate the stock prices with largest volatilities.

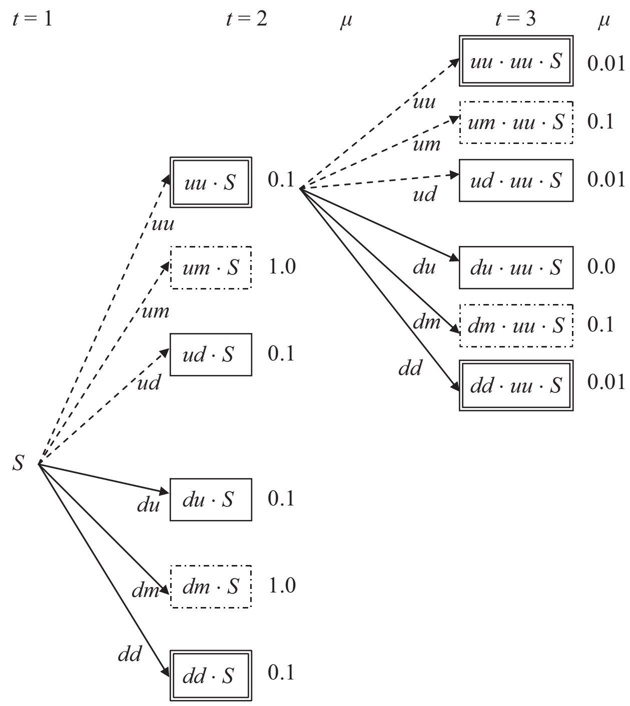

Fig. 4. Stock prices at t = 3.

Let us take another example, n = 3. There are $6^{n-1} = 6^2 = 36$ stock prices. Only stock prices derived from $S_{uu}(= uu \cdot S_{uu})$ are listed in Fig. 4. Similar to the case for n = 2, there are six stock prices derived: $uu \cdot S_{uu}$, $um \cdot S_{uu}$, $ud \cdot S_{uu}$ for the upward movement and $du \cdot S_{uu}$, $dm \cdot S_{uu}$, $dd \cdot S_{uu}$ for the downward movement. Meanwhile, $\mu(uu \cdot S_{uu}) = 0.1 \cdot 0.1 = 0.01$, $\mu(um \cdot S_{uu}) = 0.1 \cdot 1.0 = 0.1$, $\mu(ud \cdot S_{uu}) = 0.1 \cdot 0.1 = 0.01$. Similarly, $\mu(du \cdot S_{uu}) = 0.1 \cdot 0.1 = 0.01$, $\mu(dm \cdot S_{uu}) = 0.1 \cdot 1.0 = 0.1$, $\mu(dd \cdot S_{uu}) = 0.1 \cdot 0.1 = 0.01$.

2. The Process of Inference for Fuzzy Call Prices

Now we trace all the possible call prices from the stock prices. Following the definition of call prices, they are defined at $t = n$.

The call prices at $t = n$ (the last period) can be expressed as

$$C_{uu \cdot h} = \max(0, uu \cdot S_h - K) \tag{14}$$

$$C_{um \cdot h} = \max(0, um \cdot S_h - K) \tag{15}$$

$$C_{ud \cdot h} = \max(0, ud \cdot S_h - K) \tag{16}$$

$$C_{du \cdot h} = \max(0, du \cdot S_h - K) \tag{17}$$

$$C_{dm \cdot h} = \max(0, dm \cdot S_h - K) \tag{18}$$

$$C_{dd \cdot h} = \max(0, dd \cdot S_h - K) \tag{19}$$

where h represents the combination of *uu*, *um* and *ud*; *du*, *dm* and *dd*.

We can use the call prices at $t = n$ to trace all other call prices. Accordingly, the call prices from $t = n$ to $t = n - 1$ can be similarly derived.

Each fuzzy call price at $t = n - 1$ is formed as a fuzzy number (C_l, C_c, C_r), where C_l is created from the two call prices with the largest volatilities, C_c is created from the two with the medium volatilities and C_r is created from the two with the smallest volatilities. Finally, we get the general call price valuation formula as follows:

$$C_l = \frac{(a - dd) \cdot C_{uu \cdot h} + (uu - a) \cdot C_{dd \cdot h}}{(uu - dd) \cdot a} \tag{20}$$

$$C_c = \frac{(a - dm) \cdot C_{um \cdot h} + (um - a) \cdot C_{dm \cdot h}}{(um - dm) \cdot a} \tag{21}$$

$$C_r = \frac{(a - du) \cdot C_{ud \cdot h} + (ud - a) \cdot C_{du \cdot h}}{(ud - du) \cdot a} \tag{22}$$

The μ's for the derived call prices are calculated by the min operation

$$\mu_{Cq \cap C2} = \min(\mu_{C1}(p), \mu_{C2}(q)) \tag{23}$$

where $C1$ and $C2$ are the two call prices from which a new call price is derived.

For example, if we let $t = 3$, there are six call prices listed in Fig. 5.

$$C_{uu \cdot h} = \max(0, uu \cdot S_h - K)$$

$$C_{um \cdot h} = \max(0, um \cdot S_h - K)$$

$$C_{ud \cdot h} = \max(0, ud \cdot S_h - K)$$

$$C_{du \cdot h} = \max(0, du \cdot S_h - K)$$

$$C_{dm \cdot h} = \max(0, dm \cdot S_h - K)$$

$$C_{dd \cdot h} = \max(0, dd \cdot S_h - K)$$

$C_{uu \cdot h}$ and $C_{dd \cdot h}$ indicate the largest volatilities and are used to calculate the C_l at $t = 2$. Similarly, $C_{um \cdot h}$ and $C_{dm \cdot h}$ indicate the medium volatilities and are used to calculate the C_c; $C_{ud \cdot h}$ and $C_{dd \cdot h}$ indicate the smallest volatilities and are used to calculate the C_r. The μ of C_l (at $t = 2$) = min(0.01, 0.01) = 0.01. Similarly, the μ of C_c = min(0.1, 0.1) = 0.1 and the μ of C_r = min(0.01, 0.01) = 0.01.

Next, we take the call price with the greatest μ from which we derive the call price at $t = 2$, C_c. Similarly, we can calculate the 30 other call prices at $t = 3$ and the other five values of C_c, as in Fig. 6.

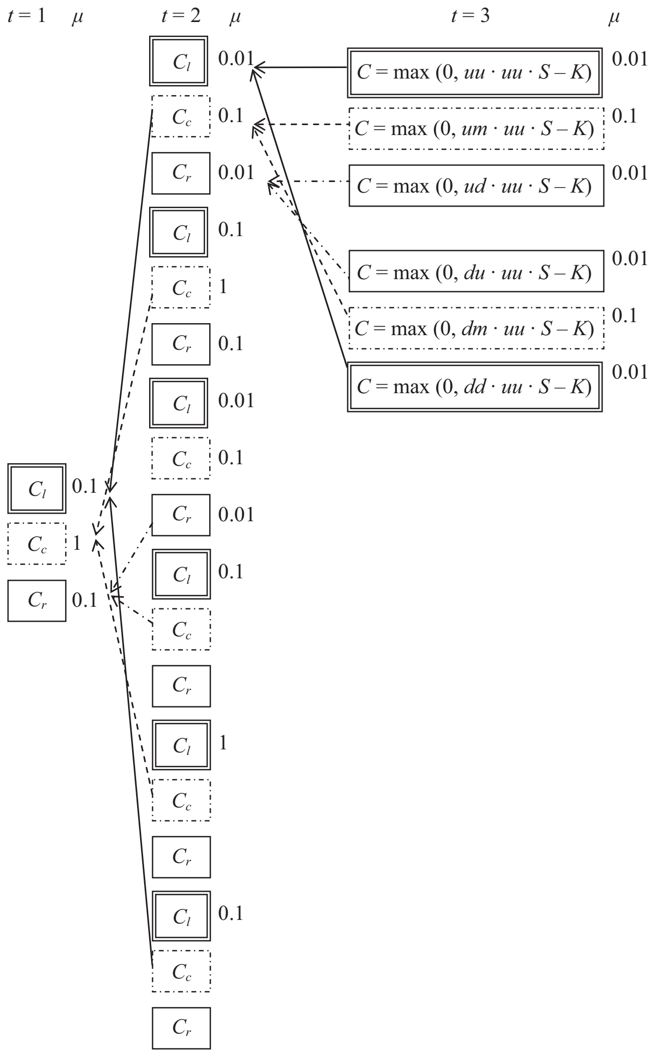

Fig. 5. Call prices at $t = 3$ and $t = 2$.

IV. EMPIRICAL ANALYSIS

Most existing studies are conceptual, which leads to the empirical findings are not able to be generalized to companies and industries. Hereafter, we utilized empirical date and analyzed the effectiveness of the proposed model.

1. Data Description

Analysis is conducted using data from the *Taiwan Economic Journal* (*TEJ*) Data Bank. The issuing date of the stock was 2000/01/11, and the maturity day was 2001/01/20. The stock price and call price at the issuing date were NT 35 and NT 9.3, respectively. The warrant we chose was Fubon 04, and its underlying security was the TCC (Taiwan Cement Corporation). In Taiwan, the date of maturity of a warrant is usually less than one year from the date of issuance. Thus we used the rate for one year time deposits from the First Commercial Bank as the risk-free rate. The rate was between 4.9% and 5%, and we chose 5% for use with the geometric mean method. The exercised price of Fubon 04 was 35 (at $t = 1$).

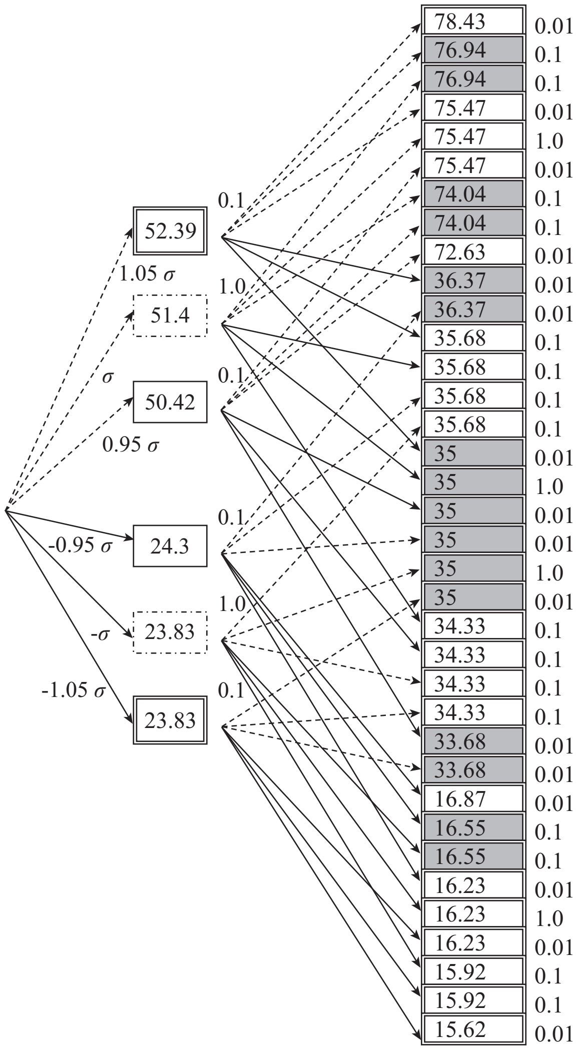

Fig. 6. Empirical example of fuzzy stock prices.

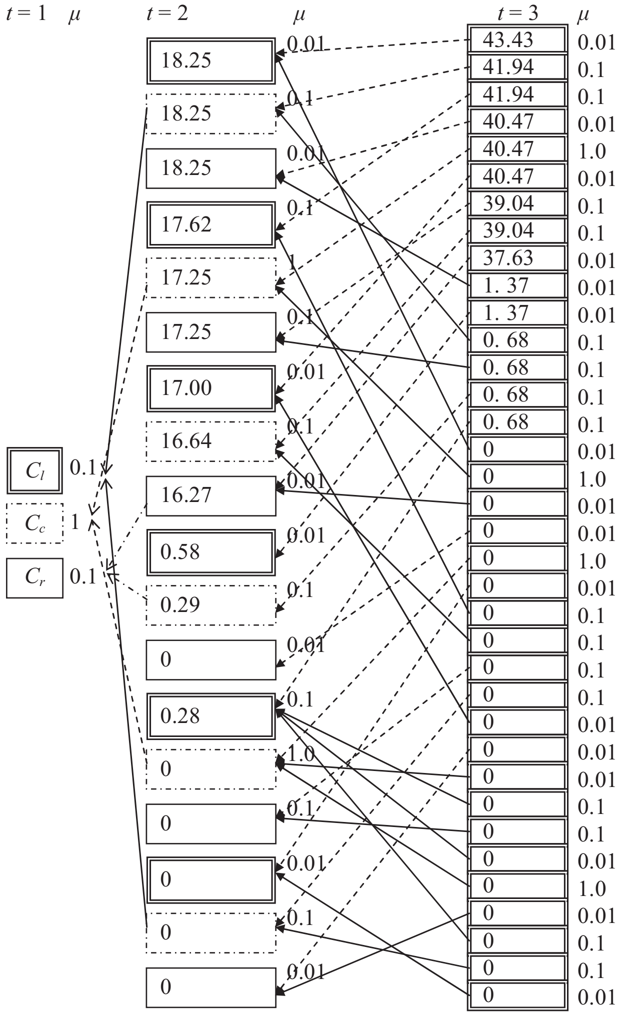

Fig. 7. Empirical example of fuzzy call prices.

Historical approach is utilized to obtain the volatilities for applying the CPR model [87]. As in Hull [45], trading days instead of calendar days were used when annualizing the Δt. The volatility ρ was estimated using the daily stock closing prices over the past 90 to 180 days.

2. Fuzzy Stock Prices

First, the estimated σ was 0.36. This was fuzzified into a fuzzy set $(\sigma(1-\rho), \sigma, \sigma(1+\rho))$. For example, if we set $\rho = 5\%$; we have a fuzzy σ of $(0.95\ \sigma, \sigma, 1.05\ \sigma)$. The fuzzy σ is calculated to be (0.34, 0.36, 0.38). Suppose the number of time periods $n = 2$. For the upward movement,

$$uu = e^{(1+\rho)\sigma\sqrt{\Delta t}},\ \text{so}\ S_{uu} = 35 \times e^{1.05\times\sigma\sqrt{0.5}} = 45.78;$$

$$um = e^{\sigma\sqrt{\Delta t}},\ \text{so}\ S_{um} = 35 \times e^{\sigma\sqrt{0.5}} = 45.20;$$

$$ud = e^{(1-\rho)\sigma\sqrt{\Delta t}},\ \text{so}\ S_{ud} = 35 \times e^{0.95\times\sigma\sqrt{0.5}} = 44.63.$$

For the downward movement,

$$du = e^{-(1-\rho)\sigma\sqrt{\Delta t}},\ \text{so}\ S_{du} = 35 \times e^{-0.95\times\sigma\sqrt{0.5}} = 27.45;$$

$$dm = e^{-\sigma\sqrt{\Delta t}},\ \text{so}\ S_{du} = 35 \times e^{\sigma\sqrt{0.5}} = 27.10;$$

$$dd = e^{-(1+\rho)\sigma\sqrt{\Delta t}},\ \text{so}\ S_{dd} = 35 \times e^{-1.05\times\sigma\sqrt{0.5}} = 26.76.$$

The details are depicted in Fig. 6.

3. Fuzzy Call Prices

From the stock price at $t = 3$ and the exercised price we compute the call prices at $t = 3$. As in Fig. 7, the call price (second value in the leftmost column) is equal to max (59.1218814-35, 0) = 24.1218814. The call price (second value in the leftmost column) is equal to max (35.45040654-35, 0) = 0.45040654. By $C_m = \dfrac{P_m \cdot C_{um} + (1-P_m)\cdot C_{dm}}{a}$, we get $C_m = 11.63459355$.

Table 1. Sensitivity analysis.

		Numbers of Periods			
		2	3	4	5
		10.03	7.67	8.28	7.92
	5%	9.60	7.35	8.16	7.75
		9.16	7.35	8.03	7.75
		10.46	7.98	8.40	8.08
	10%	9.60	7.35	8.16	7.75
ρ		8.72	7.35	7.91	7.75
		10.89	8.29	8.52	8.25
	15%	9.60	7.35	8.16	7.75
		8.27	7.35	7.79	7.75
		11.32	8.60	8.64	8.42
	20%	9.60	7.35	8.16	7.75
		7.82	7.35	7.67	7.75

Next, we choose the medium value of the triangular fuzzy number with the same weight for the operation. For instance, 11.63459355 is C_{uu} at $t = 2$; 0 is C_{dd} at $t = 2$. We thus get C_u = 5.50245741. Similarly, we can calculate the current call price of triangular fuzzy numbers as (5.50245741, 5.22736817, 5.16856623).

4. Sensitivity Analysis

To conduct sensitivity analysis, let there be four periods with fuzzy intervals ρ of 5%, 10%, 15%, and 20%. From Table 1, we know that the larger the ρ, the bigger the fuzzy interval. In other words, when ρ becomes smaller, the precision improves. In this case, the model becomes much closer to the continuous Black-Scholes model.

Since there are more nodes in the fuzzy option pricing model than in the conventional model, the speed of convergence is faster than for the previous model.

V. CONCLUSIONS

Using this combination of option pricing models and fuzzy set theory, risk averters and risk lovers can find the correct portfolio building strategy, according to the right and left values of the triangular fuzzy number, to suit their own inclinations. When market prices are lower than the left value of the triangular fuzzy number, risk lovers can buy more, risk averters can buy less. When market prices are between the right value and left value of the triangular fuzzy number, risk averters can buy less than those who are risk neutral, and risk lovers can buy much more. In other words, the study results provide more guidance for investors.

The fuzzy binomial tree option pricing model is much closer to what occurs in the real world than is the CRR model, and the concepts on which it is based are much easier to understand. Since there are three conditions for the upward and downward movements, there are six nodes for $t = 2$. We use the same method to fuzzify every node and combine them with the same value, to get 15 nodes on $t = 3$. The results are shown in Fig. 8. For the future research, the volatility can be treated as a time series. In this case, fuzzy time series model (Huarng, [41]; Song and Chissom, [77]) can be applied to forecast the volatility. Fuzzy time series models have been applied to different problem domains, such as enrollment (Huarng, [41]; Song and Chissom, [77]; Song and Chissom, [78]), stock index (Huarng, [41]; Huarng and Yu, [42]; Huarng, Yu, and Hsu, [43]), tourism (Huarng, Yu, and Parellada, [44]), etc. And some of these models have been shown to outperform their counterparts in forecasting (Song and Chissom, [77]; Song and Chissom, [78]). Hence, the application of fuzzy time series models to forecast the volatility and then to forecast the market price would be one of the interesting research topics.

		Risk lover	Risk neutral	Risk averter
r	(a)	Strong Sell	Sell	Sell
c	(b)	Sell	Sell	Hold
l	(c)	Buy	Buy	Hold
	(d)	Strong Buy	Buy	Buy

Fig. 8. Investment strategies.

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