SHAPE OPTIMIZATION OF ONE-CHAMBER PERFORATED MUFFLERS FILLED WITH WOOL USING SIMULATED ANNEALING

Min-Chie Chiu

Key words: space constraints, dissipative muffler, wool, simulated annealing.

ABSTRACT

Research on dissipative and reactive mufflers has been addressed. However, the acoustical performance —sound transmission loss (STL) —of mufflers within a constrained space is often insufficient. In this paper, to improve the acoustical efficiency, a one-chamber perforated muffler filled with sound absorbing wool optimized by using simulated annealing (SA) in conjunction with the numerical decoupling technique is presented. A numerical case in eliminating a broadband noise is also introduced. To verify the reliability of SA optimization, optimal noise abatements for the pure tone are exemplified. Before the SA operation can be carried out, the accuracy of the mathematical models has been checked using the experimental data. Results indicate that the maximal STL is precisely located at the desired target tone. Moreover, the STL can be improved when the ratio of the perforated tube’s length and the porosity of the perforated tube and the acoustical flowing resistance of the wool increase and the Mach number and the expansion ratio decrease. Consequently, a successful approach used for the optimal design of the one-chamber dissipative mufflers within a constrained space has been demonstrated.

I. INTRODUCTION

Research on mufflers used in reducing high frequency noise using a duct lined with sound absorbing material was started by Morse in 1939 [10]. Scott [17] used a volume model in solving the acoustical performance in both the circular and rectangular duct lined with porous material. Ko [7] assessed the sound transmission loss in acoustically lined flow ducts separated by porous splitters in 1975. Cummings and Chang [3] investigated the duct’s acoustical performance at various mean flows using the characteristics of bulk-reacting liners in circular ducts. On the basis of an infinite duct, the above researches analyzed the acoustical performance of the duct at a fixed duct’s diameter. Cummings and Chang [4] developed a modal method in analyzing a finite length dissipative flow duct silencer with internal mean flow in the absorbent in 1988. Regarding the volume modulus, Peat [15], in 1991, used a transfer matrix in evaluating the acoustical performance for an absorption silencer element. Selamet et al. [18, 19] assessed the acoustical attenuation for perforated concentric absorbing silencers and hybrid silencers using a one-dimensional analytical method, a three-dimensional boundary element method (BEM), and an experimental study. Wang proposed a three-dimensional boundary element method (BEM) in analyzing the acoustical performance of a one-chamber dissipative muffler [24]. By way of plane wave theory, Munjal [12], in 2003, proposed a four-pole transfer matrix in solving the sound attenuation of pod silencers lined with porous material. Xu et al. [25] assessed the sound attenuation in dissipative expansion chambers using the characteristic equation in 2004. However, the assessment of a muffler’s optimal shape design within a constrained space was rarely tackled.

In previous work [1, 2], the shape optimization of one-chamber mufflers equipped with a perforated resonating tube within a constrained situation has been discussed. However, its acoustical performance is insufficient in dealing with a higher and broader frequency noise because of the characteristic narrow band effect. To improve acoustical efficiency, an assessment of one-chamber perforated mufflers lined with porous material is presented.

An assessment of a perforated acoustical element used to depress low frequency sound energy was introduced and discussed by Sullivan and Crocker in 1978 [22]. On the basis of the coupled differential equations, a series of theoretical and numerical techniques in decoupling the acoustical problems have been proposed [5, 13, 16, 20, 21, 23]. Peat [14] publicized a successfully numerical decoupling method by finding the eigen value in transfer matrices. Here, the numerical
decoupling methods used in forming a four-pole system matrix are in line with the simulated annealing (SA) method. These, in turn, are responsible for developing a new muffler shape by adjusting the geometric parameters (i.e., the length of the dissipative muffler, the porosity of the inner duct, the diameter of the perforated hole on the inner duct, and the diameter of the inner duct) and the acoustical property of the wool (i.e., the wool’s acoustic flowing resistance) within certain space constraints. To appreciate the acoustical performance with/without sound absorbing wool, a one-chamber perforated muffler without sound absorbing wool is also numerically assessed.

II. THEORETICAL BACKGROUND

In this paper, two one-chamber perforated mufflers with and without sound absorbing material were adopted for the noise abatement in the constrained blower room shown in Fig. 1. The outlines and recognition of acoustical elements and the related acoustic pressure $p$ and acoustic particle velocity $u$ for various mufflers is depicted in Fig. 2.

As indicated in Fig. 2(a), the one-chamber perforated muffler (muffler A) filled with wool and composed of three acoustical elements is identified as having two categories of components — two straight ducts (I) and one perforated dissipative chamber (II). Fig. 2(b) indicates that the one-chamber muffler (muffler B) hybridized with a perforated resonating tube composed of three acoustical elements is identified as having two categories of components — two straight ducts (I) and one perforated resonating tube (II). The detailed mathematical derivation of various muffler systems is presented below.

1. A One-chamber Muffler with Sound Absorbing Material (Muffler A)

As shown in Appendix A, the transfer matrix of a perforated chamber filled with sound absorbing wool is derived. The individual matrixes with respect to straight ducts and a perforated resonating tube are described as follows [1, 2]:

$$
\begin{bmatrix}
\bar{p}_1 \\
\rho_c \bar{u}_1
\end{bmatrix} = e^{-j\mu M_{\lambda L} (\lambda - 1)} \begin{bmatrix}
TS_{11,1} & TS_{11,2} \\
TS_{12,1} & TS_{12,2}
\end{bmatrix} \begin{bmatrix}
\bar{p}_2 \\
\rho_c \bar{u}_2
\end{bmatrix}
$$

(1)

$$
\begin{bmatrix}
\bar{p}_2 \\
\rho_c \bar{u}_2
\end{bmatrix} = \begin{bmatrix}
TPD_{12,1} & TPD_{12,2}
\end{bmatrix} \begin{bmatrix}
\bar{p}_3 \\
\rho_c \bar{u}_3
\end{bmatrix}
$$

(2)

$$
\begin{bmatrix}
\bar{p}_3 \\
\rho_c \bar{u}_3
\end{bmatrix} = e^{-j\mu M_{\lambda L} (\lambda - 1)} \begin{bmatrix}
TS_{13,1} & TS_{13,2} \\
TS_{12,1} & TS_{12,2}
\end{bmatrix} \begin{bmatrix}
\bar{p}_4 \\
\rho_c \bar{u}_4
\end{bmatrix}
$$

(3)

The total transfer matrix assembled by multiplication is

$$
\begin{bmatrix}
\bar{p}_1 \\
\rho_c \bar{u}_1
\end{bmatrix} = e^{-j\mu M_{\lambda L} (\lambda - 1)} \begin{bmatrix}
TS_{11,1} & TS_{11,2} \\
TS_{12,1} & TS_{12,2}
\end{bmatrix} \begin{bmatrix}
TPD_{12,1} & TPD_{12,2}
\end{bmatrix} \begin{bmatrix}
\bar{p}_3 \\
\rho_c \bar{u}_3
\end{bmatrix} \begin{bmatrix}
\bar{p}_4 \\
\rho_c \bar{u}_4
\end{bmatrix}
$$

(4)
Subsequently, a simplified form of a system matrix for the muffler is expressed as

\[
\begin{pmatrix}
\bar{p}_1 \\
\rho_p c \bar{p}_1
\end{pmatrix}
= \Pi \begin{bmatrix}
T_w(f) \\
\rho_p c \bar{p}_2
\end{bmatrix}
\]

(5)

The sound transmission loss (STL) of the muffler A is expressed as [11]

\[
STL(Q, f, RT_1, RT_2, RT_3, RT_4, RT_5)
= 20 \log \left( \frac{1}{2} (T_{11} + T_{12} + T_{21} + T_{22}) \right) + 10 \log \left( \frac{S_I}{S_4} \right)
\]

(6a)

where

\[
RT_i = L_i / L_o; RT_2 = \sigma_f; RT_3 = d_H; RT_4 = \eta; RT_5 = d_i / D_o
\]

(6b)

2. A One-chamber Muffler without Sound Absorbing Material (Muffler B)

As derived in the previous work [1, 2], individual transfer matrixes with respect to straight ducts and a perforated resonating tube are described as follows:

The total transfer matrix assembled by multiplication is

\[
\begin{pmatrix}
\bar{p}_1 \\
\rho_p c \bar{p}_1
\end{pmatrix}
= e^{-\frac{\rho_p c \bar{p}_1}{\gamma M_a + S_p}}
\begin{pmatrix}
TS11_{1,1} & TS11_{1,2} \\
TS11_{2,1} & TS11_{2,2}
\end{pmatrix}
\begin{pmatrix}
TPR12_{1,1} & TPR12_{1,2} \\
TPR12_{2,1} & TPR12_{2,2}
\end{pmatrix}
\begin{pmatrix}
TS13_{3,1} & TS13_{3,2} \\
TS13_{2,1} & TS13_{2,2}
\end{pmatrix}
\begin{pmatrix}
\bar{p}_4 \\
\rho_p c \bar{p}_4
\end{pmatrix}
\]

(7)

2. A One-chamber Muffler without Sound Absorbing Material (Muffler B)

As derived in the previous work [1, 2], individual transfer matrixes with respect to straight ducts and a perforated resonating tube are described as follows:

The total transfer matrix assembled by multiplication is

\[
\begin{pmatrix}
\bar{p}_1 \\
\rho_p c \bar{p}_1
\end{pmatrix}
= \Pi \begin{bmatrix}
T_w(f) \\
\rho_p c \bar{p}_2
\end{bmatrix}
\]

(8)

The sound transmission loss (STL) of the muffler B is expressed as [11]

\[
STL(Q, f, RT_1', RT_2', RT_3', RT_4', RT_5')
= 20 \log \left( \frac{1}{2} (T_{11} + T_{12} + T_{21} + T_{22}) \right) + 10 \log \left( \frac{S_I}{S_4} \right)
\]

(9)

3. Overall Sound Power Level

The overall SWL subjected to the muffler at the outlet is

\[
SWL_I = 10 \log \left( \sum_m 10^{SWL(f_m)_{-STL(f_m)/10}} \right)
\]

(10)

where (1) SWL(f_m) is the original SWL at the inlet of the muffler (or pipe outlet), and m is the index of the octave band frequency.

(2) STL(f_m) is the muffler’s STL with respect to the relative octave band frequency.

4. Objective Function

By using the formulas of Eqs. (6) and (9), the objective function used in the SA optimization with respect to each type of muffler was established.

For a one-chamber perforated muffler with sound absorbing wool, the objective function in maximizing the STL at the pure tone (f) is

\[
OBJ_{11} = STL(f, RT_1, RT_2, RT_3, RT_4, RT_5)
\]

(11)

The objective function in eliminating the overall SWL is

\[
OBJ_{12} = SWL_I (RT_1, RT_2, RT_3, RT_4, RT_5)
\]

(12)

Similarly, for a one-chamber perforated muffler hybridized without sound absorbing wool, the objective function in maximizing the STL at the pure tone (f) is

\[
OBJ_{21} = STL(f, RT_1', RT_2', RT_3', RT_4', RT_5')
\]

(13)

The objective function in eliminating the overall SWL is

\[
OBJ_{22} = SWL_I (RT_1, RT_2, RT_3, RT_4, RT_5)
\]

(14)

III. MODEL CHECK

Before performing the SA optimal simulation on mufflers, an accuracy check of the mathematical model on the acoustical elements of one-chamber perforated mufflers with in the chamber are performed using the experimental data from Lee [8]. As depicted in Fig. 3, the theoretical and experimental data are accurate and in agreement. Therefore, the proposed fundamental mathematical models are acceptable. Consequently, the model linked with the numerical method is applied to the shape optimization in the following section.

IV. CASE STUDIES

In this paper, a muffler confined inside a blower room is shown in Fig. 1. The primary noise of the blower’s sound power level inside the pipe outlet (muffler’s inlet) is listed in Table 1. To efficiently reduce the sound energy, two kinds of mufflers (muffler A: a one-chamber perforated muffler with sound absorbing wool; muffler B: a one-chamber muffler without sound absorbing wool) are adopted. As shown in Fig. 1, the available space for a muffler is 0.3 m in width, 0.3 m in height, and 1.0 m in length. To simplify the optimization,
of heating and keeping a metal at a stabilized temperature with the minimum possible energy. Annealing is the process of bringing the system from an arbitrary initial state to a state of some physical system, the internal energy of the system in that state. Therefore, the goal is to bring the system from an arbitrary initial state to a state with the minimum possible energy. Annealing is the process of heating and keeping a metal at a stabilized temperature while cooling it slowly. Slow cooling allows the particles to keep their state close to the minimal energy state. The algorithm starts by generating a random initial solution. The scheme of SA is a variation of the hill-climbing algorithm. All downhill movements for improvement are accepted for the decrement of the system’s energy. In order to escape from the local optimum, SA also allows movement resulting in solutions that are worse (uphill moves) than the current solution. To imitate the evolution of the SA algorithm, a new random transition property (pb(X')) which results in a higher energy condition will then be accepted; otherwise, it is rejected. Each successful substitution of the new current solution will lead to the decrement of the current temperature as

$$ T_{new} = kk * T_{old} $$

where $C$ and $T$ are the Boltzmann constant and the current temperature. Moreover, compared with the new random probability of rand(0,1), if the transition property (pb(X')) is greater than a random number of rand(0,1), the new solution will be acknowledged as the new current solution with the transition property (pb(X') of 1. If the change is not negative (AF 0), the probability of making the transition to the new state $X'$ will be a function $pb(AF/CT)$ of the energy difference $AF = (X') - F(X)$ between the two states and a function of the global time-varying parameter $T$. The new transition property (pb(X')) varied from 0-1 will be calculated by Boltzmann’s factor (pb(X') = $\exp(-AF/CT)$ as shown in Eq. (15)

$$ pb(X') = \begin{cases} 1, & AF \leq 0 \\ \exp(-AF/CT), & AF > 0 \end{cases} $$

$$ AF = F(X') - F(X) $$

where $C$ and $T$ are the Boltzmann constant and the current temperature. Moreover, compared with the new random probability of rand(0,1), if the transition property (pb(X')) is greater than a random number of rand(0,1), the new solution (worse solution) which results in a higher energy condition will then be accepted; otherwise, it is rejected. Each successful substitution of the new current solution will lead to the decay of the current temperature as

$$ T_{new} = kk * T_{old} $$

where $kk$ is the cooling rate.

The flow diagram of the SA optimization is described and

---

**Table 1. Unsilenced primary SWL of a blower inside a duct outlet.**

<table>
<thead>
<tr>
<th>Frequency - Hz</th>
<th>500</th>
<th>1000</th>
<th>2000</th>
<th>overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>SWLO - dB(A)</td>
<td>98</td>
<td>110</td>
<td>123</td>
<td>123</td>
</tr>
</tbody>
</table>

---

**Table 2. The corresponding space constraints and the ranges of design parameters for a muffler.**

<table>
<thead>
<tr>
<th>Range of design parameters</th>
<th>muffler with</th>
<th>muffler without</th>
</tr>
</thead>
<tbody>
<tr>
<td>Targeted f; [1700] (Hz)</td>
<td>1.0 (m); $D_o$ = 0,3 (m);</td>
<td>Targeted f; [1700] (Hz); $Q = 0.01$ (m$^3$/s);</td>
</tr>
<tr>
<td>sound absorption $L_o$ = 1.0 (m); $D_o$ = 0,3 (m);</td>
<td>$RT_1$: [0.2, 0.8]; $RT_2$: [4000, 20000];</td>
<td></td>
</tr>
<tr>
<td>wool (Muffler A) $RT_1$: [0.00175, 0.007]; $RT_2$: [0.03, 0.1];</td>
<td>$RT_1$: [0.4, 0.8];</td>
<td></td>
</tr>
<tr>
<td>$RT_2$: [0.03, 0.1]; $RT_2$: [0.00175, 0.007];</td>
<td>$RT_2$: [0.4, 0.8]; $RT_2$: [0.2, 0.8]; $RT_2$: [0.2, 0.8]</td>
<td></td>
</tr>
<tr>
<td>wool (Muffler B) $RT_2$: [0.2, 0.8];</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 3. Performance of a single-chamber dissipative muffler without the mean flow [$L = 0.2572$ (m); $d1 = 0.00049$ (m); $d2 = 0.001644$ (m); $q = 8.4$%; $dH = 0.0049$ (m); density = 100 (kg/m$^3$); $M = 0$] [Experimental data is from Lee [8]].**
Table 3. Optimal design data for muffler A at various SA parameters (iter and kk) (targeted tone at 1700 Hz).

<table>
<thead>
<tr>
<th>SA parameter</th>
<th>iter</th>
<th>kk</th>
<th>$RT_1$</th>
<th>$RT_2$</th>
<th>$RT_3$</th>
<th>$RT_4$</th>
<th>$RT_5$</th>
<th>STL (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50</td>
<td>0.91</td>
<td>0.3948</td>
<td>9195</td>
<td>0.003455</td>
<td>0.05273</td>
<td>0.5299</td>
<td>19.95</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.93</td>
<td>0.4690</td>
<td>11170</td>
<td>0.004104</td>
<td>0.06138</td>
<td>0.5793</td>
<td>29.14</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.95</td>
<td>0.4811</td>
<td>11500</td>
<td>0.004209</td>
<td>0.06279</td>
<td>0.5874</td>
<td>31.01</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.97</td>
<td>0.5003</td>
<td>12010</td>
<td>0.004377</td>
<td>0.06503</td>
<td>0.6002</td>
<td>34.26</td>
</tr>
<tr>
<td>50</td>
<td>0.99</td>
<td>0.5371</td>
<td>12990</td>
<td>0.0047</td>
<td>0.06933</td>
<td>0.6247</td>
<td>41.86</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.99</td>
<td>0.5643</td>
<td>13720</td>
<td>0.004938</td>
<td>0.07251</td>
<td>0.6429</td>
<td>49.33</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.99</td>
<td>0.6388</td>
<td>15700</td>
<td>0.00559</td>
<td>0.0812</td>
<td>0.6926</td>
<td>72.35</td>
</tr>
<tr>
<td>1000</td>
<td>0.99</td>
<td>0.6207</td>
<td>15220</td>
<td>0.005431</td>
<td>0.07908</td>
<td>0.6805</td>
<td>74.27</td>
<td></td>
</tr>
</tbody>
</table>

Table 4. Optimal design data for two kinds of mufflers (targeted tone at 1700 Hz) (iter = 1000; kk = 0.99).

<table>
<thead>
<tr>
<th>Muffler type</th>
<th>$RT_1$</th>
<th>$RT_2$</th>
<th>$RT_3$</th>
<th>$RT_4$</th>
<th>$RT_5$</th>
<th>STL (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muffler A</td>
<td>0.6207</td>
<td>15220</td>
<td>0.005431</td>
<td>0.07908</td>
<td>0.6805</td>
<td>74.27</td>
</tr>
<tr>
<td>Muffler B</td>
<td>0.0826</td>
<td>0.0023</td>
<td>0.4997</td>
<td>0.5271</td>
<td>0.5401</td>
<td>53.61</td>
</tr>
</tbody>
</table>

Notes: $RT_1^* = \eta; RT_2^* = dH; RT_3^* = d/D_o; RT_4^* = (L_2 + 2 \cdot L_4)/L_4; RT_5^* = L_3/(2 \cdot L_4 + L_2)$

VI. RESULTS AND DISCUSSION

1. Results

The accuracy of the SA optimization depends on the cooling rate (kk) and the number of iterations (iter). To achieve good optimization, both the cooling rate (kk) and the number of iterations (iter) are varied step by step

$$kk = (0.91, 0.93, 0.95, 0.97, 0.99); \text{iter} = (50, 100, 500, 1000)$$

The results of two kinds of optimizations (one, a pure tone noise; the other, a broadband noise) are described as follows:

1) Pure Tone Noise Optimization

By using Eqs. (6) and (11), the maximization of the STL with respect to muffler A at the specified higher pure tone (1700 Hz) was performed first. As indicated in Table 3, eight sets of SA parameters are tried in the muffler’s optimization.

Obviously, the optimal design data can be obtained from the last set of SA parameters at (kk, iter) = (0.99, 1000). Similarly, by using the SA parameters of (kk = 0.99, iter = 1000) in mufflers A and B, the resultant STLs at a targeted tone (1700 Hz) have been summarized in Table 4. Using these optimal design data in a theoretical calculation, the resultant curves of the STL with respect to various mufflers are plotted in Fig. 5.

2) Broadband Noise Optimization

By using the formulas of Eqs. (10), (12), and (14) and the SA parameters of (kk = 0.99, iter = 1000), the optimization...
process for minimizing the sound power level at the muffler’s outlet within a limited space is performed. The optimal result at various mufflers is obtained and summarized in Table 5. Using these optimal design data in a theoretical calculation, the resultant curves of the SWL with respect to various mufflers are plotted in Fig. 6. As illustrated in Table 5, the resultant SWL with respect to muffler A and B will be reduced from 123 dB(A) to 78.51 dB and 97.52 dB.

2. Discussion

To achieve a sufficient optimization, the selection of the appropriate SA parameter set is essential. As indicated in Table 3, the best SA set with respect to muffler A (a one-chamber perforated muffler with sound absorbing wool) at the targeted pure tone of 1700 Hz has been shown. Table 4 indicates that for a fixed targeted frequency (1700 Hz), the acoustical performance will be improved when the length of the dissipative muffler, the porosity of the inner duct, the diameter of the perforated hole on the inner duct, the diameter of the inner duct, and the acoustical flowing resistance increase. As revealed in Fig. 5, the STL is precisely maximized at the desired frequency when using the best design data. Moreover, it is obvious that the acoustical performance of muffler A (with sound absorbing wool) is superior to that of the muffler B (without sound absorbing wool) in the higher frequency of 1700 Hz.

In dealing with a higher frequency noise emitted from a blower, the optimal design of two kinds of mufflers within a limited space has been discussed and shown in Table 5 and Fig. 6. As indicated in Table 5, the noise reduction with respect to muffler A and muffler B is 44.49 dB and 25.48 dB. Moreover, as indicated in Fig. 6, muffler A having the widest band and highest STL is superior to muffler B in the higher frequency region. As indicated in Figs. 5 and 6, muffler B having narrow-band STL characteristics will be suitable for eliminating the pure tone and lower frequency noise. However, in dealing with a broadband’s noise, muffler B is insufficient. It has been seen that the overall acoustical performance will be substantially improved using muffler A where the sound absorbing wool is added. Moreover, the acoustical performance will be improved when the acoustical flowing resistance is increased.

To appreciate the influence of the STL with respect to other parameters such as Mach number \( M \), diameter of the perforated hole \( d_H \), porosity \( \eta \), ratio of the perforated tube’s length \( L_{2}/L_o \), expansion ratio \( d_1/D_o \), and the acoustical flowing resistance of the wool \( \sigma_{fr} \) for a one-chamber perforated muffler filled with wool, the trend of the STL profiles for an optimized muffler at 1700 Hz has been calculated by varying the value of each parameter \( M, d_H, \eta, L_{2}/L_o, d_1/D_o, \) and \( \sigma_{fr} \). As indicated in Figs. 7-9, it is obvious that the STL is inversely proportional to the Mach number \( M \) & the

### Table 5. Optimal design data for two kinds of mufflers (broadband noise) \((\text{iter} = 1000; \text{kk} = 0.99)\).

<table>
<thead>
<tr>
<th>Muffler type</th>
<th>Design parameters</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muffler A</td>
<td>( RT_1 ) ( RT_2 ) ( RT_3 ) ( RT_4 ) ( RT_5 )</td>
<td>SWL (dB)</td>
</tr>
<tr>
<td>Muffler A</td>
<td>0.7581 18880 0.003633 0.09511 0.7721</td>
<td>78.51</td>
</tr>
<tr>
<td>Muffler B</td>
<td>( RT'_1 ) ( RT'_2 ) ( RT'_3 ) ( RT'_4 ) ( RT'_5 )</td>
<td>SWL (dB)</td>
</tr>
<tr>
<td>Muffler B</td>
<td>0.0775 0.0037 0.4077 0.6259 0.7228</td>
<td>97.52</td>
</tr>
</tbody>
</table>
Fig. 7. The STL with respect to frequencies at various $M$.

Fig. 8. The STL with respect to frequencies at various $d/d_o$ ($=RT_3$).

Fig. 9. The STL with respect to frequencies at various $L_2/L_o$ ($=RT_4$).

Fig. 10. The STL with respect to frequencies at various $dH$ ($=RT_5$).

Fig. 11. The STL with respect to frequencies at various $\eta$ ($=RT_4$).

Fig. 12. The STL with respect to frequencies at various $\sigma_{fr}$ ($=RT_2$).
expansion ratio \((d_i/D_o)\) and proportional to the ratio of the perforated tube’s length \((L_i/L_o)\). Additionally, Fig. 10 indicates that the peak frequency will decrease when the diameter of the perforated hole \((dH)\) increases. However, as indicated in Fig. 11, the peak frequency will increase when the porosity of the perforated tube \((\eta)\) increases. Moreover, as indicated in Fig. 12, the peak frequency will shift to the left and the related \(STL\) will increase when the acoustical flowing resistance of the wool \((\sigma_f)\) increases.

Consequently, Mach number \((M)\), diameter of the perforated hole \((dH)\), porosity \((\eta)\), ratio of the perforated tube’s length \((L_i/L_o)\), expansion ratio \((d_i/D_o)\), and the flowing resistance of the wool \((\sigma_f)\) play essential roles in eliminating the noise level in mufflers.

VII. CONCLUSION

It has been shown that the one-chamber perforated mufflers with/without sound absorbing wool in conjunction with a SA optimizer can be easily and efficiently optimized under space limits by using a numerical decoupling technique, plane wave theory, as well as a four-pole transfer matrix. As indicated in Table 3, eight kinds of SA parameters \((kk, iter)\) play essential roles in the solution’s accuracy during SA optimization. As indicated in Fig. 5, the \(STL\) is precisely maximized at the desired frequency; therefore, the tuning ability established by adjusting the design parameters of the mufflers is reliable. As indicated in Figs. 5 and 6 muffler B having narrow-band \(STL\) characteristics will be suitable for eliminating the pure tone and lower frequency noise. However, in dealing with a broadband’s noise, muffler B is insufficient. It has been seen that the overall acoustical performance will be substantially improved using muffler A where the sound absorbing wool is added. Moreover, the acoustical performance will be improved when the acoustical flowing resistance of the wool \((\sigma_f)\) & the ratio of the perforated tube’s length \((L_i/L_o)\) & the porosity \((\eta)\) are increased and the Mach number \((M)\) & the expansion ratio \((d_i/D_o)\) are decreased. Furthermore, the peak frequency of the \(STL\) will decrease when the diameter of the perforated hole \((dH)\) increases. Consequently, muffler A having sound absorbing wool which results in the widest band and highest \(STL\) is superior to muffler B. This approach used for the optimal design of the \(STL\) proposed in this study is easy and quite effective.

NOMENCLATURE

This paper is constructed on the basis of the following notations:

\[ C_i \] coefficients in function \((\Gamma_i = C_i e^{\gamma i})\)
\[ C_v \] sound speed \((m/s)\)
\[ \tilde{C_v} \] sound speed in a wool \((m/s)\)
\[ dH \] the diameter of a perforated hole on the i-th inner tube \((m)\)
\[ D_i \] diameter of the i-th perforated tubes \((m)\)
\[ D_o \] diameter of the outer tube \((m)\)
\[ f \] cyclic frequency \((Hz)\)
\[ \text{iter} \] maximum iteration
\[ j \] imaginary unit
\[ k \] wave number \((= \omega/c_o)\)
\[ \tilde{k} \] the wave number for the wool
\[ kk \] cooling rate in SA
\[ L_i \] total length of the muffler \((m)\)
\[ M \] mean flow Mach number
\[ OBJ \] objective function \((dB)\)
\[ p \] acoustic pressure \((Pa)\)
\[ p_{i} \] acoustic pressure at the i-th node \((Pa)\)
\[ pb(T) \] transition probability
\[ Q \] volume flow rate of venting gas \((m^3 s^{-1})\)
\[ S_i \] section area at the i-th node \((m^2)\)
\[ STL \] sound transmission loss \((dB)\)
\[ SWL_o \] unsilenced sound power level inside the muffler’s inlet \((dB)\)
\[ SWL_T \] overall sound power level inside the muffler’s output \((dB)\)
\[ t_i \] the thickness of the i-th inner perforated tube \((m)\)
\[ TS_{ij} \] components of four-pole transfer matrices for an acoustical mechanism with straight ducts
\[ TPR_{ij} \] components of a four-pole transfer matrix for an acoustical mechanism with an empty perforated chamber
\[ TPD_{ij} \] components of a four-pole transfer matrix for an acoustical mechanism with a perforated chamber filled with sound absorbing wool
\[ T_{ij}^{c} \] components of a four-pole transfer system matrix acoustic particle velocity \((m/s)\)
\[ u \] acoustic particle velocity at the i-th node \((m/s)\)
\[ u_{i} \] acoustical particle velocity passing through a perforated hole from the i-th node to the j-th node \((m/s)\)
\[ v_i \] mean flow velocity at the inner perforated tube \((m/s)\)
\[ v_2 \] mean flow velocity at the outer tube \((m/s)\)
\[ \rho_0 \] air density \((kg m^{-3})\)
\[ \tilde{\rho}_o \] wool density \((kg m^{-3})\)
\[ \rho_f \] acoustical density at the i-th node
\[ \xi \] specific acoustical impedance of the i-th inner perforated tube
\[ \eta \] the porosity of the i-th inner perforated tube.
\[ \alpha_c \] the structure factor for the wool
\[ \sigma_f \] the acoustical flowing resistance for the wool
\[ \gamma \] i-th eigen value of \([N]_{i\times4}\)
\[ [\Omega]_{4\times4} \] the model matrix formed by four sets of eigen vectors \(\Omega_{\text{eq}}\) of \([N]_{4\times4}\)
ACKNOWLEDGMENTS

The author acknowledges the financial support of the National Science Council (NSC99-2221-E-235-001, Taiwan, ROC).

APPENDIX A - TRANSFER MATRIX OF A PERFORATED CHAMBER FILLED WITH SOUND ABSORBING WOOL

As indicated in Fig. 1, the perforated resonator is composed of an inner perforated tube and an outer resonating chamber. Based on Sullivan and Crocker’s derivation [22], the continuity equations and momentum equations with respect to inner and outer tubes at nodes 2 and 2 are listed below.

**Inner tube:**

**continuity equation:**
\[ V \frac{\partial \rho_2}{\partial x} + \rho_2 \frac{\partial u_2}{\partial x} + \frac{4 \rho_2}{D_1} \frac{\partial u_2}{\partial t} + \frac{\partial \rho_2}{\partial t} = 0 \]  \hspace{1cm} (A1)

**momentum equation:**
\[ \rho_2 \left( \frac{\partial}{\partial t} + V \frac{\partial}{\partial x} \right) u_2 + \frac{\partial \rho_2}{\partial x} = 0 \]  \hspace{1cm} (A2)

**Outer tube:**

**continuity equation:**
\[ \rho \frac{\partial u_{2a}}{\partial x} - \frac{4 D_i \rho}{D_o^2 - D_i^2} \frac{\partial u_{2a}}{\partial t} + \frac{\partial \rho_{2a}}{\partial t} = 0 \]  \hspace{1cm} (A3)

**momentum equation:**
\[ \rho \frac{\partial u_{2a}}{\partial t} + \frac{\partial \rho_{2a}}{\partial x} = 0 \]  \hspace{1cm} (A4)

Assuming that the acoustic wave is a harmonic motion
\[ P(x, t) = P(x) \cdot e^{i\omega t} \]  \hspace{1cm} (A5)

under the isentropic processes in ducts, it yields
\[ P(x) = \rho(x) \cdot c_o^2 \]  \hspace{1cm} (A6)

Assuming that the perforation along the inner tube is uniform \( (d \xi dx = 0) \), the acoustic impedance of the perforation \( (\rho_o c_o \xi) \) is
\[ \rho_o c_o \xi = \frac{p_o(x) - p_{2a}(x)}{u(x)} \]  \hspace{1cm} (A7)

where \( \xi \) is the specific acoustical impedance of the perforated tube.

The empirical formulations developed by Sullivan [22] and Rao [16] for the perforates with and without mean flow are adopted in this study.

For perforates with stationary medium, we have
\[ \xi = [0.006 + jk(t + 0.75dH)]/\eta_{h, A} \]  \hspace{1cm} (A8a)

For perforates with grazing flow, we have
\[ \xi_{h, A} = [0.514 D_i M_i/(L_o \eta) + j0.95k(t + 0.75dH)]/\eta \]  \hspace{1cm} (A8b)

where \( dH \) is the diameter of a perforated hole on inner tube, \( t \) is the thickness of an inner perforated tube, and \( \eta \) is the porosity of the perforated tube.

Plugging Eqs. (A5)-(A7) into Eqs. (A1)-(A4) and Eliminating \( u_2 \) and \( u_{2a} \), we have
\[ \left( 1 - M^2 \right) \frac{d^2}{dx^2} - 2 jMk \frac{d}{dx} + k^2 \right] P_2 = 0 \]  \hspace{1cm} (A9)

\[ \left[ \frac{d^2}{dx^2} + \tilde{k}^2 \right] P_{2a} + j \frac{4kD_i \tilde{\rho}}{(D_i^2 - D_o^2)} \left[ P_2 - P_{2a} \right] = 0 \]  \hspace{1cm} (A10)

where \( M_i = \frac{V_o}{c_o} \)

Alternatively, Eqs. (A9) and (A10) can be expressed as
\[ p_2 + \alpha_1 p_2 + \alpha_2 p_2 + \alpha_3 p_{2a} + \alpha_4 p_{2a} = 0 \]  \hspace{1cm} (A11a)

\[ \alpha_1 p_2 + \alpha_5 p_2 + \alpha_6 p_{2a} + \alpha_7 p_{2a} + \alpha_8 p_{2a} = 0 \]  \hspace{1cm} (A11b)

where

\[ \alpha_1 = - \frac{j M}{1 - M^2} \left( 2k - j \frac{4}{D_i \xi} \right); \alpha_2 = \frac{1}{1 - M^2} \left( k^2 - j \frac{4k}{D_i \xi} \right); \]

\[ \alpha_3 = \frac{M}{1 - M^2}; \alpha_4 = \frac{j}{1 - M^2}; \alpha_5 = \frac{4k}{D_i \xi}; \alpha_6 = 0; \]

\[ \alpha_7 = \frac{j 4kD_i \tilde{\rho}}{(D_i^2 - D_o^2)}; \alpha_8 = 0; \]

\[ \alpha_9 = \frac{\omega}{c}; \alpha_{10} = \tilde{k} - \frac{j 4kD_i \tilde{\rho}}{(D_i^2 - D_o^2)}; k = \frac{\omega}{c} \]  \hspace{1cm} (A11c)

Let...
\[ p_2 = \frac{dp_2}{dx} = y_1, \quad p_{2A} = \frac{dp_{2a}}{dx} = y_2, \quad p_2 = y_3, \quad p_{2A} = y_4 \] (A12)

According to Eqs. (A11) and (A12), the new matrix between \( \{y'\} \) and \( \{y\} \) is

\[
\begin{bmatrix}
    y_1 \\
    y_2 \\
    y_3 \\
    y_4
\end{bmatrix} = \begin{bmatrix}
    -\alpha_1 & -\alpha_3 & -\alpha_2 & -\alpha_4 \\
    1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
    y_1 \\
    y_2 \\
    y_3 \\
    y_4
\end{bmatrix}
\] (A13a)

which can be briefly expressed as

\[ \{y'\} = [N]\{y\} \] (A13b)

Let

\[ \{y\} = [\Omega]\{\Gamma\} \] (A14a)

which is

\[
\begin{bmatrix}
    \frac{dp_2}{dx} \\
    \frac{dp_{2a}}{dx} \\
    p_2 \\
    p_{2a}
\end{bmatrix} = \begin{bmatrix}
    \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} \\
    \Omega_{21} & \Omega_{22} & \Omega_{23} & \Omega_{24} \\
    \Omega_{31} & \Omega_{32} & \Omega_{33} & \Omega_{34} \\
    \Omega_{41} & \Omega_{42} & \Omega_{43} & \Omega_{44}
\end{bmatrix} \begin{bmatrix}
    \Gamma_1 \\
    \Gamma_2 \\
    \Gamma_3 \\
    \Gamma_4
\end{bmatrix}
\] (A14b)

\([\Omega]_{4x4}\) is the model matrix formed by four sets of eigen vectors \( \Omega_{ax} \) of \([N]_{4x4}\).

Combining Eq. (A14) with Eq. (A13) and then multiplying \([\Omega]\)^{-1} by both sides yields

\[ [\Omega]^{-1}[\Omega][\Gamma] = [\Omega]^{-1}[N][\Omega][\Gamma] \] (A15a)

Set

\[ [\chi] = [\Omega]^{-1}[N][\Omega] = \begin{bmatrix}
    \chi_1 & 0 & 0 & 0 \\
    0 & \chi_2 & 0 & 0 \\
    0 & 0 & \chi_3 & 0 \\
    0 & 0 & 0 & \chi_4
\end{bmatrix} \] (A15b)

where \( \chi \) is the eigen value of \([N]\)

Eq. (A13) can be thus rewritten as

\[ \{\Gamma\} = [\chi][\{\Gamma\}] \] (A16)

Obviously, Eq. (A15) is a decoupled equation. The related solution obtained is

\[ \Gamma_i = C_i e^{\lambda_i} \] (A17)

Plugging Eq. (A17) into Eq. (A14b) and doing a rearrangement, we have

\[
\begin{bmatrix}
    p_2(x) \\
    p_{2a}(x) \\
    dp_2(x) \\
    dp_{2a}(x)
\end{bmatrix} = \begin{bmatrix}
    \Omega_{11} e^{\lambda_1} & \Omega_{12} e^{\lambda_2} & \Omega_{13} e^{\lambda_3} & \Omega_{14} e^{\lambda_4} \\
    \Omega_{21} e^{\lambda_1} & \Omega_{22} e^{\lambda_2} & \Omega_{23} e^{\lambda_3} & \Omega_{24} e^{\lambda_4} \\
    \Omega_{31} e^{\lambda_1} & \Omega_{32} e^{\lambda_2} & \Omega_{33} e^{\lambda_3} & \Omega_{34} e^{\lambda_4} \\
    \Omega_{41} e^{\lambda_1} & \Omega_{42} e^{\lambda_2} & \Omega_{43} e^{\lambda_3} & \Omega_{44} e^{\lambda_4}
\end{bmatrix} \begin{bmatrix}
    C_1 \\
    C_2 \\
    C_3 \\
    C_4
\end{bmatrix}
\] (A18)

From Eqs. (A2) and (A4), it becomes

\[
\rho \dot{c} u_z = -\frac{1}{jk + M \gamma} \frac{dp_2}{dx}; \quad \dot{\rho} \dot{c} u_z = -\frac{1}{jk} \frac{dp_{2a}}{dx} \] (A19)

Plugging Eq. (A19) into Eq. (A18) yields

\[
\begin{bmatrix}
    p_2(x) \\
    p_{2a}(x) \\
    \rho \dot{c} u_z(x) \\
    \dot{\rho} \dot{c} u_z(x)
\end{bmatrix} = \begin{bmatrix}
    H_{11} & H_{12} & H_{13} & H_{14} \\
    H_{21} & H_{22} & H_{23} & H_{24} \\
    H_{31} & H_{32} & H_{33} & H_{34} \\
    H_{41} & H_{42} & H_{43} & H_{44}
\end{bmatrix} \begin{bmatrix}
    C_1 \\
    C_2 \\
    C_3 \\
    C_4
\end{bmatrix} \] (A20)

Substituting two cases of \( x = 0 \) and \( x = L_c \) into Eq. (A20) yields

\[
\begin{bmatrix}
    p_2(0) \\
    p_{2a}(0) \\
    \rho \dot{c} u_z(0) \\
    \dot{\rho} \dot{c} u_z(0)
\end{bmatrix} = [A] \begin{bmatrix}
    p_2(L_c) \\
    p_{2a}(L_c) \\
    \rho \dot{c} u_z(L_c) \\
    \dot{\rho} \dot{c} u_z(L_c)
\end{bmatrix}
\] (A21a)

where

\[ [A] = [H(0)][H(L_c)]^{-1} \] (A21b)

The boundary conditions for the inner tube are

\[
\begin{align*}
    p_{2a}(0) & = -j \dot{\rho} \dot{c} \cot(k L_a) \quad \text{at } x = 0 \\
    p_{2a}(L_c) & = -j \dot{\rho} \dot{c} \cot(k L_b) \quad \text{at } x = L_c
\end{align*}
\] (A22a)

(Continued...)

\[
\begin{bmatrix}
\bar{p}_2 \\
\rho_c \bar{c}_n \bar{n}_4
\end{bmatrix}
= \begin{bmatrix}
TPD_{x,1} & TPD_{x,2} \\
TPD_{x,3} & TPD_{x,4}
\end{bmatrix}
\begin{bmatrix}
\bar{p}_3 \\
\rho_c \bar{c}_n \bar{n}_4
\end{bmatrix}
\]  

(A23b)

where

\[
\bar{p}_2 = p_2(0); \bar{u}_2 = u_2(0); \bar{p}_3 = p_{2a}(L_C); \bar{u}_3 = u_{2a}(L_C);
\]  

(A23c)

REFERENCES