FORCED CONVECTION FILM CONDENSATION OF DOWNWARD-FLOWING VAPOR ON HORIZONTAL TUBE WITH WALL SUCTION EFFECT

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Key words: film condensation, flowing vapor, potential flow, wall suction.

ABSTRACT

The heat transfer performance of the condensate layer formed by a downward-flowing, dry saturated vapor flowing over an isothermal horizontal tube with wall suction effects is examined. Under the effects of wall suction, the gravity force, the potential flow pressure gradient, and the interfacial shear stress, the local/mean Nusselt numbers of the condensate film are obtained as a function of the Jakob number $Ja$, the Prandtl number $Pr$, the Rayleigh number $Ra$, the two-phase mean Reynolds number $Re_v$, the dimensionless pressure gradient parameter $P$, and the suction parameter $Sw$. The results show that the mean Nusselt number increases with increasing $Re_v$ and $Sw$. Furthermore, the dimensionless pressure gradient parameter has a negligible effect on the Nusselt number for $Re_v < 2 \times 10^6$, but has a negative effect on the Nusselt number for $Re_v \geq 2 \times 10^6$.

I. INTRODUCTION

The problem of forced convection film condensation occurs in a variety of industrial applications and has attracted many researchers’ interests. The condensate layer flows over the contact surface under the effects of gravity, the interfacial vapor shear stress, and the pressure gradient, and may lead to an accumulation of liquid in certain regions depending on the profile of the surface. In 1916, Nusselt [15] introduced the concept of local balance of viscous forces and the weight of the condensate film and showed that the heat transfer in condensation depends on local film thickness. The results showed that the heat transfer in a condensate layer is directly related to the local film thickness. Since that time, many other researchers have also examined the laminar film condensation of quiescent vapors. For example, Rohsenow [16], Sparrow and Gregg [18], Chen [3], Koh [12], Koh et al. [13], Denny and Mills [4], and Merte [14] improved the accuracy of Nusselt’s original laminar film condensation solutions by removing the overly-restrictive assumptions, such as the interfacial shear stress, the convective and inertial effects were negligible, and the temperature within the condensate layer varied linearly with the film thickness.

Condensation on horizontal tubes has many thermal engineering applications, ranging from heat exchange systems to chemical engineering processes, air-conditioning equipment, and so on. The problem of laminar condensation on a horizontal tube was first analyzed by Sparrow and Gregg [18] in 1959. Gaddis [6] used a series expansion method to solve the coupled boundary layer equations for laminar film condensation on a horizontal cylinder. Neglecting the inertial and convective effects in the condensate film, Honda and Fujii [8] formulated the problem of forced flow condensation on a horizontal cylinder as a conjugate heat transfer problem. Yang and Chen [21] investigated the role of surface tension and ellipticity in laminar film condensation on a horizontal elliptical tube. The results showed that the mean heat transfer coefficient obtained using an elliptical tube with its major axis oriented in the vertical direction was greater than that obtained from a simple circular tube.

When a vapor travels over a horizontal tube at high velocity, the heat transfer problem becomes a forced convection film condensation problem, and the analysis must take account of the interfacial vapor shear force. Shekirladze and Gomelauri [17] showed that the surface shear stress depends on the momentum transferred to the condensate film by the suction mass. Fujii et al. [5] derived two-phase boundary layer equations for laminar film condensation on a horizontal cylinder. Hsu and Yang [9] proposed a combined forced and natural convection film condensation model to examine the
effects of the pressure gradient and the wall temperature on the condensate layer produced by a vapor stream flowing in a downward direction over the surface of a horizontal tube. It was shown that the mean Nusselt number decreased with an increasing wall temperature variation in the forced convection region, but was insensitive to the wall temperature in the natural convection region. More recently, Hu and Chen [10] analyzed the problem of turbulent film condensation on an inclined elliptical tube, and showed that for lower vapor velocities, a higher eccentricity increases the heat transfer over the upper half of the tube, but reduces the heat transfer over the lower half of the tube. In addition, it was shown that the heat transfer performance of the condensate film improved with an increasing vapor velocity.

Many previous studies [1, 2, 19, 21] have reported that the condensate heat transfer performance can be improved by applying a wall suction effect. For the problem of condensation on the outer surface of a horizontal tube, an inner suction tube is needed to carry out the liquid condensate which sucks from the outer surface of the tube. Accordingly, the present study examines the forced convection heat transfer performance of a condensate layer flowing in the downward direction over the external surface of a horizontal tube with suction force effects acting at the tube surface. The analyses take account of the effects of both the potential flow pressure gradient and the interfacial shear stress. In general, it is shown that the hydrodynamic characteristics of the condensate film are significantly dependent on both the wall suction effect and the interfacial shear stress effect.

II. THE TRANSCEIVER STRUCTURE

Consider a horizontal, clean, permeable tube with a radius \( R \) and a constant temperature \( T_w \) immersed in a downward flowing pure vapor. Assume that the vapor is at its saturation temperature \( T_{sat} \) and has a uniform velocity \( U_\infty \). If \( T_w \) is lower than \( T_{sat} \), a thin condensate layer is formed on the surface of the tube and runs downward over the tube in the peripheral direction under the combined effects of the gravity force, the pressure gradient and the interfacial shear stress. Under steady-state conditions, the thickness of the liquid film, \( \delta \), has a minimum value at the top of the tube and increases gradually as the liquid flows in the downward direction.

The physical model and coordinate system considered in the present study are shown in Fig. 1, in which the curvilinear coordinates \((x, y)\) are aligned along the surface of the tube and the surface normal, respectively. In analyzing the heat transfer characteristics of the condensate film, the same set of assumptions as those used by Rohsenow [16] are applied, namely (1) the condensate film flow is steady and laminar, and thus the effects of inertia and convection are negligible and can be ignored (i.e. a creeping film flow is assumed); (2) the wall temperature, vapor temperature and properties of the dry vapor and condensate, respectively, are constant, and (3) the condensate film has negligible kinetic energy. Consequently, the continuity, momentum and energy conservation equations for the liquid film are as given as follows:

**Continuity equation**

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{1}
\]

**Momentum equation in x-direction**

\[
0 = \mu \frac{\partial^2 u}{\partial y^2} + (\rho - \rho_v) g \sin \theta - \frac{dp}{Rd\theta}. \tag{2}
\]

**Energy equation**

\[
0 = \alpha \frac{\partial^2 T}{\partial y^2}. \tag{3}
\]

The boundary conditions are given by:

(1) At \( y = 0 \), \( u = 0 \), \( T = T_w \) \tag{4}

(2) At \( y = \delta \), \( \frac{\partial u}{\partial y} = \frac{\tau_u}{\mu} \), \( T = T_{sat} \) \tag{5}

Assuming that the condensate film thickness is negligible compared to the tube radius, the pressure gradient term in Eq. (2) can be derived by applying the Bernoulli equation. In other words, Eq. (2) can be rewritten as

\[
\mu \frac{\partial^2 u}{\partial y^2} = -(\rho - \rho_v) g \sin \theta - \rho_v u \frac{du}{Rd\theta}. \tag{6}
\]
According to potential flow theory, for a uniform vapor flow traveling at a velocity $U_\infty$ over a circular cylinder, the tangential vapor velocity at the edge of the boundary layer is given by

$$u_t = U_\infty 2 \sin \theta .$$  \hspace{1cm} (7)

The pressure gradient can then be obtained as

$$\rho u_t \frac{du_t}{Rd\theta} = \frac{4 \rho U_\infty^2 \sin 2\theta}{D} .$$ \hspace{1cm} (8)

The boundary condition for the interfacial shear stress (Eq. (5)) can be simplified using the Shekriladze and Gomelauri [17] model as

$$\tau_\delta = m'' u_t = 2m'' U_\infty \sin \theta ,$$ \hspace{1cm} (9)

where $m''$ is the condensate mass flux. Neglecting the sensible heat of subcooling, the energy equation for steady laminar flow is written as

$$m'' h_\delta = k \frac{\partial T}{\partial y} = \frac{k \Delta T}{\delta} ,$$ \hspace{1cm} (10)

where $\Delta T = T_{sw} - T_w$.

Substituting Eqs. (9) and (10) into Eq. (5), the interfacial shear stress is obtained as

$$\frac{\partial u}{\partial y} = \frac{2k \Delta T U_\infty \sin \theta}{h_\delta u_\delta} = k \frac{\Delta T}{\delta} \text{ at } y = \delta .$$ \hspace{1cm} (11)

Integrating the momentum equation given in Eq. (6) with Eq. (8), and applying the boundary conditions given in Eqs. (4) and (11), the velocity distribution equation can be derived as

$$u = \frac{2k \Delta T U_\infty \sin \theta y}{h_\delta \mu} + \frac{(\rho - \rho_t) R \sin \theta + 4\rho U_\infty^2 \sin 2\theta}{\mu} \left( \delta y - \frac{1}{2} y^2 \right) .$$ \hspace{1cm} (12)

According to Nusselt’s classical analysis [15], an energy balance exists on each element of the liquid film with height $\delta$ and width $dx$, i.e.

$$\frac{d}{dx} \left[ \int_0^\delta \rho u (h_\delta + C_p (T_{sw} - T)) dy \right] dx$$

$$+ \rho (h_\delta + C_p \Delta T) v_c dx = k \frac{\Delta T}{\delta} dx .$$ \hspace{1cm} (13)

Substituting Eq. (12) into Eq. (13), introducing dimensionless parameters $Ja$, $Pr$, $Ra$, $Re_v$, $Re_w$, $P$, $Sw$, and the dimensionless liquid film thickness parameter, $\delta^* = \delta/R$, then applying the relation $dx = Rd\theta$ yields the following expression for the condensate film thickness:

$$\delta^* \frac{d}{d\theta} \left[ 3 \frac{Ja}{Ra} Re_v \sin \theta + \delta^* \left( \sin \theta + 2 Re_v^2 P \sin 2\theta \right) \right]$$

$$+ 3 \delta^* Sw \delta^* = \frac{3 Ja}{Ra} .$$ \hspace{1cm} (14)

where $R$ is the tube radius, $Ja$ is the Jakob number, $Pr$ is the Prandtl number, $Ra$ is the Rayleigh number, $Re_v$ is the two-phase mean Reynolds number, $Re_w$ is the suction Reynolds number, $P$ is the dimensionless pressure gradient parameter, and $Sw$ is the suction parameter. $Ja$, $Pr$, $Ra$, $Re_v$, $Re_w$, $P$ and $Sw$ are defined as follows:

$$Ja = \frac{C_p \Delta T}{h_{\delta e} \frac{3}{8} C_p \Delta T} ,$$

$$Pr = \frac{\mu C_p \Delta T}{h_{\delta e}} ,$$

$$Ra = \frac{d (\rho - \rho_t) g \rho R^3}{\mu^2} ,$$

$$Re_v = \frac{\rho U_\infty R}{\mu} ,$$

$$S_{\nu} = \left( 1 + \frac{5 Ja}{8} \right) Re_v \frac{Pr}{Ra} .$$ \hspace{1cm} (15)

The corresponding boundary condition is given as

$$\frac{d \delta^*}{d\theta} = 0 \text{ at } \theta = 0 .$$ \hspace{1cm} (16)

The first term inside the differentiation brackets in Eq. (14) results from the interfacial shear stress, while the term involving $P$ describes the effect of the pressure gradient produced by the potential flow. Meanwhile, the term involving $Sw$ represents the effect of the wall suction. Note that when these three terms are omitted, Eq. (14) reduces to the pure natural convection film condensation problem (i.e. Nusselt type condensation problem).

For computational convenience, Eq. (14) can be rewritten as

$$3 \frac{Ja}{Ra} Re_v \delta^* \frac{d \delta^*}{d\theta} \left( \sin \theta + \delta^* \cos \theta \right)$$

$$+ 3 \delta^* \frac{d \delta^*}{d\theta} \left( \sin \theta + 2 Re_v^2 P \sin 2\theta \right)$$

$$+ \delta^* \left( \cos \theta + 4 Re_v^2 P \cos 2\theta \right) + 3 Sw \delta^* = \frac{3 Ja}{Ra} .$$ \hspace{1cm} (17)

The numerical solution procedure commences by substi-
tuting the boundary condition given in Eq. (16), i.e. \( \frac{d\delta}{d\theta}_{\theta=0} = 0 \), into Eq. (17). The following polynomial equation with respect to the dimensionless condensate film thickness at \( \theta = 0 \), \( \delta^*_0 \), can then be obtained:

\[
(1 + 4Re^2Pr)\delta_0^{*4} + 3\frac{Ja}{Ra}Re, \delta_0^{*3} + 3Sw\delta_0^* = 3\frac{Ja}{Ra}.
\] (18)

Obviously, the dimensionless liquid film thickness cannot have a negative value, and thus in solving Eq. (18), the film thickness is constrained by \( \delta^*_0 \geq 0 \). Note that the exact value of \( \delta^*_0 \) can be derived using the bisection method [11(a)]. Substituting the derived value of \( \delta^*_0 \) into Eq. (17), the variation of \( \delta^* \) in the \( \theta \) direction can be calculated using the forward difference shooting method [11(b)]. In accordance with Nusselt’s theory [15], the dimensionless local heat transfer coefficient (i.e. the local Nusselt number) can be calculated as

\[
Nu_\theta = \frac{h_\theta D}{k},
\] (19)

where

\[
h_\theta = \frac{k}{\delta}.
\]

Substituting \( h_\theta \) into Eq. (19), the local Nusselt number can be rewritten as

\[
Nu_\theta = \frac{2}{\delta^*(\theta)}.
\] (20)

Meanwhile, the mean Nusselt number is given by

\[
\overline{Nu} = \frac{1}{\pi} \int_0^\pi Nu_\theta d\theta.
\] (21)

III. RESULTS AND DISCUSSIONS

In simulating practical engineering problems, the physical parameters used in Eq. (17) (i.e. \( R, Ja, Pr, Ra, Re, P, \) and \( Sw \)) must be assigned reasonable values. The present analyses assume the working liquid to be water-vapor and use the dimensional and dimensionless parameter values presented in Table 1 (reproduced from [7]).

Figs. 2(a) and 2(b) show the distributions of the dimensionless film thickness and local Nusselt number along the surface of the tube as a function of the two-phase Reynolds number \( Re \) given constant wall suction parameters of \( Sw = 0 \) and \( 10^{-10} \), respectively. (Note that the remaining parameters are assigned the characteristic values shown in Table 1, i.e. \( Ja = 0.02, Ra = 2 \times 10^{11} \) and \( P = 5.5 \times 10^{-15} \)). As shown, the dimensionless film thickness has a minimum value at the top of the tube and increases with increasing \( \theta \). This is to be expected since the current analyses consider the case of
falling film condensation, and thus the effects of gravity minimize the film thickness on the upper surface of the horizontal tube, but cause the film thickness to increase toward an infinite value at the lower surface of the tube. Moreover, it can be seen that the dimensionless liquid film thickness decreases, while the local Nusselt number increases, with an increasing value of $Re_e$. The physical reason for this is that a higher value of $Re_e$ implies a stronger interfacial shear force, with the result that enhance the liquid condensate falls along the tube circumference. As a result, the thickness of the liquid film is reduced and a steeper temperature gradient is formed. Finally, comparing Figs. 2(a) and 2(b), it can be seen that the Nusselt number increases with an increasing wall suction parameter $Sw$ due to the corresponding reduction in the condensate layer thickness.

Figs. 3(a) and 3(b) show the effects of the dimensionless pressure gradient parameter, $P$, on the variation of the dimensionless film thickness and local Nusselt number along the surface of the tube for wall suction parameters of $Sw = 0$ and $10^{-10}$, respectively. (Note that $Ja = 0.02$, $Ra = 2 \times 10^{11}$ and $Re_e = 3.4 \times 10^4$ in both cases.) It is seen that for both values of $Sw$, the thickness of the liquid film on the upper surface of the tube decreases with increasing $P$. In other words, the liquid film thickness at the upper surface of the tube decreases when the effects of the potential flow pressure gradient are taken into account. Conversely, on the lower surface of the tube, the liquid film thickness increases with a higher $P$. In other words, the liquid film thickness at the lower surface of the tube increases when the effects of the potential flow pressure gradient are taken into account. These findings are reasonable since the potential flow pressure gradient has a positive value on the upper surface of the tube, but has a negative value on the lower surface of the tube. From Eq. (14), it can be seen that for a positive value of the pressure gradient (i.e. $P \sin 2\theta > 0$), the liquid film is pushed down over the side surfaces of the tube, and thus a reduction in the film thickness occurs. Conversely, for a negative value of the pressure gradient (i.e. $P \sin 2\theta < 0$), the liquid film is pushed in the upward direction, causing an accumulation of the condensate and a corresponding increase in the thickness of the condensate layer.

Fig. 4 illustrates the variation of the mean Nusselt number, $\overline{Nu}$, with $Re_e$ as a function of the dimensionless pressure gradient parameter, $P$, and the wall suction parameter, $Sw$. It can be seen that the mean Nusselt number increases with increasing $Re_e$ and increasing $Sw$. In addition, it is observed that for lower values of the two-phase Reynolds number (i.e. $Re_e < 2 \times 10^5$), the dimensionless pressure gradient parameter, $P$, has a negligible effect on the variation of $\overline{Nu}$ with $Re_e$.
on the mean Nusselt number. However, $\bar{Nu}$ decreases with increasing $P$ at higher values of $Re_v$ (i.e. $Re_v \geq 2 \times 10^6$). The greater effect of $P$ on $\bar{Nu}$ at higher values of $Re_v$ is to be expected since a higher value of $Re_v$ corresponds to a reduced liquid film thickness on the upper surface of the tube, and hence a corresponding increase in the positive effects of the dimensionless pressure gradient parameter $P$.

### IV. CONCLUSION

This study has investigated the forced convection heat transfer performance of a condensate layer flowing in the downward direction over a horizontal tube with wall suction effects. The results have shown that the gravity effect causes the condensate layer thickness to have a minimum value on the upper surface of the tube but to increase toward an infinite value on the lower surface of the tube. In addition, it has been shown that the interfacial shear stress causes a reduction in the thickness of the liquid film, and therefore increases the heat transfer performance. Moreover, the potential flow pressure gradient enhances the heat transfer coefficient on the upper surface of the tube, but reduces the heat transfer coefficient on the lower surface of the tube. Finally, the results have shown that the effects of the potential pressure gradient on the mean Nusselt number must be taken into account when the two-phase Reynolds number has a value greater than $Re_v = 2 \times 10^6$.

#### NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$C_p$</td>
<td>specific heat at constant pressure</td>
</tr>
<tr>
<td>$D$</td>
<td>diameter of circular tube</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration of gravity</td>
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<tr>
<td>$h$</td>
<td>heat transfer coefficient</td>
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<tr>
<td>$h_{lv}$</td>
<td>heat of vaporization</td>
</tr>
<tr>
<td>$J_a$</td>
<td>Jakob number defined in Eq. (15)</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity</td>
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<tr>
<td>$K$</td>
<td>permeability of porous medium</td>
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<tr>
<td>$m^*$</td>
<td>condensate mass flux</td>
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<tr>
<td>$Nu$</td>
<td>Nusselt number defined in Eq. (19)</td>
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<tr>
<td>$p$</td>
<td>static pressure of condensate</td>
</tr>
<tr>
<td>$P$</td>
<td>dimensionless pressure gradient parameter defined in Eq. (15)</td>
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<tr>
<td>$Pr$</td>
<td>Prandtl number defined in Eq. (15)</td>
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<td>$R$</td>
<td>radius of circular tube</td>
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<td>$Ra$</td>
<td>Rayleigh number defined in Eq. (15)</td>
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<td>$Re_v$</td>
<td>two-phase mean Reynolds number defined in Eq. (15)</td>
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<td>$Re_w$</td>
<td>suction Reynolds number defined in Eq. (15)</td>
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<td>$Sw$</td>
<td>suction parameter defined in Eq. (15)</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>saturation temperature minus wall temperature</td>
</tr>
<tr>
<td>$u$</td>
<td>velocity component in x-direction</td>
</tr>
<tr>
<td>$u_t$</td>
<td>tangential vapor velocity at edge of boundary layer defined in Eq. (7)</td>
</tr>
<tr>
<td>$U_\infty$</td>
<td>vapor velocity of free stream</td>
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<tr>
<td>$\nu$</td>
<td>velocity component in y-direction</td>
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####Greek symbols

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<tr>
<th>Symbol</th>
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<tr>
<td>$\delta$</td>
<td>condensate film thickness</td>
</tr>
<tr>
<td>$\mu$</td>
<td>liquid viscosity</td>
</tr>
<tr>
<td>$\rho$</td>
<td>liquid density</td>
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<tr>
<td>$\rho_v$</td>
<td>vapor density</td>
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<td>$\alpha$</td>
<td>thermal diffusivity</td>
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<td>$\theta$</td>
<td>angle measured from top of tube</td>
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<tr>
<td>$\tau_s$</td>
<td>interfacial shear stress</td>
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#### Superscripts

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<tr>
<td>*</td>
<td>average quantity</td>
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<tr>
<td>$^*$</td>
<td>dimensionless variable</td>
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#### Subscripts

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<tr>
<td>sat</td>
<td>saturation property</td>
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<td>$w$</td>
<td>quantity at wall</td>
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#### ACKNOWLEDGMENTS

This study was supported by the National Science Council of Taiwan (NSC 99-2221-E-218-013).

#### REFERENCES


