Journal of Marine Science and Technology, Vol. 25, No. 3, pp. 276-282 (2017)
DOI: 10.6119/JMST-016-1216-2

INTEGRATED DESIGN FOR ROBUST STABILIZATION OF UNCERTAIN SWITCHED NONLINEAR SYSTEMS

Jian-Qiang Wang[1] and Chee-Fai Yung[1, 2]

Key words: switched nonlinear systems, control Lyapunov functions, state feedback, robust stability.

ABSTRACT

An integrated design of switching laws and feedback controllers for uncertain switched nonlinear control-affine systems with an arbitrary number of subsystems is developed. Sufficient conditions for the existence of globally asymptotically stabilizing state feedback laws are derived using the control Lyapunov function approach. Additionally, an explicit rule for constructing switching laws and formula for synthesizing feedback controllers are presented. An illustrative example confirmed the feasibility of the theoretical results.

I. INTRODUCTION

The study on switched systems is motivated by the existence of practical control systems that cannot be asymptotically stabilized using a single smooth feedback control law (Brockett, 1983). Moreover, previous studies have demonstrated that switched systems can be used to describe a wide range of physical and engineering systems (Zefran and Burdick, 1998; Dayawansa and Martin, 1999; Liberzon, 2003). The majority of previous studies on switched systems have focused on stability analysis (Ye et al., 1998; Dayawansa and Martin, 1999; Hespanha and Morse, 1999; Liberzon et al., 1999; Liberzon and Morse, 1999; Agrachev and Liberzon, 2001; Zhao and Dimirovski, 2004; Zhao and Hill, 2008; Lin and Antsaklis, 2009). One paper proved that a switched system is asymptotically stable under arbitrary switching if and only if there exists a common Lyapunov function for all subsystems (Liberzon, 2003). If the switching signal is restricted such that the switching interval is longer than a *dwell time*, the switched system is asymptotically stable if all subsystems are asymptotically stable and the dwell time is sufficiently long (e.g., Hespanha and Morse, 1999).

When considering controller synthesis of switched control systems, previous studies have mainly focused on linear subsystems (Zefran and Burdick, 1998; Daafouz et al., 2002; Xu and Antsaklis, 2004). Almost all related results have been derived in terms of linear matrix inequalities. For switched nonlinear control systems in some particular forms (e.g., strict-feedback, lower triangular, feedforward, and p-normal forms), numerous designs have been proposed for stabilizing or L_2-gain rendering controllers or switching rules (Wu, 2008; Ma and Zhao, 2010; Long and Zhao, 2011a, 2011b; Hou et al., 2013). Backstepping-based approaches were employed in these studies for constructing common control Lyapunov functions and synthesizing stabilizing controllers. However, these methods cannot be applied to solve control problems of switched nonlinear control-affine system.

Relatively few studies have been conducted on switched nonlinear (control-affine) systems in some particular forms. Yang et al. (2009, 2011) discussed the design of stabilizing switching rules for switching nonlinear systems with unstable subsystems. In one study (Wu, 2009), a common control Lyapunov function (CLF) approach was proposed to derive the necessary and sufficient conditions for the existence of stabilizing controllers for switched nonlinear control-affine systems with *arbitrary switching* between two nonlinear control-affine subsystems. An explicit formula for constructing uniformly stabilizing controllers was also provided. Another study (Mhaskar et al., 2005) developed a Lyapunov-based model predictive control scheme for achieving constrained stabilization in switched nonlinear control-affine systems with a prescribed switching schedule.

To date, only a few methods have been proposed for the integrated design of switching rules and feedback controllers to stabilize switched nonlinear control-affine systems. In the study of El-Farra et al. (2005), a family of bounded nonlinear state feedback controllers was initially designed to enforce asymptotic stability in individual closed-loop modes. A set of switching laws was then designed to orchestrate switching between subsystems and thus guarantees asymptotic stability. In that approach, all subsystems required their own CLFs (El-Farra et al., 2005). Moreover, the presented switching rules "implicitly" determined the times when switching between subsystems were

Paper submitted 06/07/16; revised 09/14/16; accepted 12/16/16. Author for correspondence: Chee-Fai Yung (e-mail: yung@mail.ntou.edu.tw, cfyung@ntu.edu.tw).
[1] *Electrical Engineering Department, National Taiwan Ocean University, Keelung, Taiwan, R.O.C.*
[2] *Department of Mathematics, National Taiwan University, Taipei, Taiwan, R.O.C.*

permissible. Explicit switching rules were not provided. Wang and Yung (2016) proposed an integrated design of switching laws and feedback controllers for uncertain switched nonlinear systems with only two subsystems. Sufficient conditions for the existence of globally asymptotically stabilizing state feedback laws were derived. However, how to extend the proposed results to switched systems with more than two subsystems was not discussed.

In this study, we design integrated switching rules and feedback controllers for achieving the global asymptotic stability of uncertain switched nonlinear control-affine systems with an arbitrary number of subsystems (in contrast with the two subsystems considered in Wang and Yung (2016)). The uncertainties we considered in this study were more general than those considered in Wang and Yung (2016). Based on the CLF approach (Artstein, 1983; Sontag, 1983, 1989; Krstic et al., 1995), we developed a robust switched CLF method to derive sufficient conditions for the existence of stabilizing feedback control laws (switching rules + feedback controllers). The conditions proposed in this paper for the two subsystem case are quite different from those proposed in Wang and Yung (2016). Moreover, our approach does not require the subsystems to have their own CLFs as is the case in El-Farra et al. (2005) method. Individual closed-loop modes can be unstable. An explicit switching rule and formula for synthesizing feedback controllers are also presented. To the best of our knowledge, similar approaches have not yet been reported.

Notation:

$\overline{S}$ is the closure of S;
∂S is the boundary of S; and
$B_\varepsilon \equiv \left\{x \in R^n \,\middle|\, \|x\| \le \varepsilon\right\}$.

II. PROBLEM FORMULATION AND PRELIMINARIES

This section presents the formulation of the problem to be solved and outlines the concept of CLFs.

1. Problem Formulation

Consider an uncertain switched control system with two nonlinear subsystems:

$$\begin{aligned}&\dot{x}(t) = f_{\sigma(t)}(x(t)) + \Delta f_{\sigma(t)}(x(t)) + g_{\sigma(t)}(x(t))u(t),\\ &\sigma(t) \in \{1,\ 2,\ ...,\ N\}\end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state of the system; $u(t) \in R^m$ is the control input; $f_i : R^n \to R^n$ and $g_i : R^n \to R^{n\times m}$, $i = 1, 2, \ldots, N$, are known smooth functions; and $\sigma : [0, \infty) \to \{1, 2, \ldots, N\}$ is the switching signal to be designed. Suppose that the uncertain terms satisfy $\Delta f_i(x) = \Gamma_i F_i(x)$, $i = 1, 2, \ldots, N$, where $\Gamma_i \in R^{n\times r}$ is an uncertain matrix satisfying $\|\Gamma_i\| \le 1$ and $F_i : R^n \to R^r$ is a known vector function. Without loss of generality, assume that $f_i(0) = 0$ and $F_i(0) = 0$, $i = 1, 2, \ldots, N$. Fig. 1 presents the system diagram.

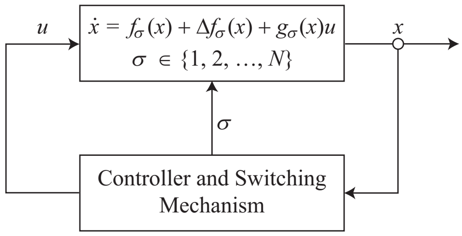

Fig. 1. System diagram.

The design objective is to find a switching rule σ and N continuous functions $p_1, p_2, \ldots, p_N : R^n \to R^m$ such that the origin is the globally asymptotically stable equilibrium of the closed-loop system

$$\dot{x} = f_\sigma(x) + \Delta f_\sigma(x) + g_\sigma(x)p_\sigma(x),\ \sigma \in \{1, 2, \ldots, N\} \tag{2}$$

for all possible uncertainties.

2. CLFs

To solve the problem in question, we first outline the concept of a CLF. Consider the following (nonswitched) nonlinear control system:

$$\dot{x} = f(x) + g(x)u\,. \tag{3}$$

Definition 1 (Sontag, 1989): A smooth positive definite and radially unbounded function $V : R^n \to R_0^+$ is a CLF of the system described by (3) if, for each $x \in R^n \setminus \{0\}$,

$$\inf_{u\in R^m}\left\{\frac{\partial V(x)}{\partial x} f(x) + \frac{\partial V(x)}{\partial x} g(x)u\right\} < 0\,. \qquad \square$$

The existence of CLFs is a necessary and sufficient condition for the existence of globally asymptotically stabilizing controllers for the system described by (3). To ensure the existence of *continuous* stabilizing feedback laws, we still need the CLF to satisfy the *small control property* (SCP).

Definition 2 (Sontag, 1989): A CLF $V : R^n \to R_0^+$ of system (3) is said to satisfy the SCP if for each $\varepsilon > 0$ there is a $\delta > 0$ such that, if $0 \ne 0$ and $\|x\| < \delta$, there is some u with $\|u\| < \varepsilon$ such that

$$\frac{\partial V(x)}{\partial x} f(x) + \frac{\partial V(x)}{\partial x} g(x)u < 0\,. \qquad \square$$

Sontag (1989) proved that if there is a CLF V satisfying the SCP for system (3), then a continuous stabilizing feedback law can be obtained.

We now review a useful lemma that is used later in this paper.

Lemma 1 (Petersen, 1987): Given any positive definite matrix function $Q(x)$ and any matrix functions $M(x)$ and $N(x)$ of compatible dimensions, the inequality

$$2x^T M(x)\Gamma N(x)x \le x^T M(x)Q(x)M^T(x)x + x^T N^T(x)Q^{-1}(x)N(x)x$$

holds for any Γ (of compatible dimension) satisfying $\|\Gamma\| \le 1$. □

III. MAIN RESULTS

This section proposes the integrated design of switching rules and feedback controllers for stabilizing an uncertain switched nonlinear control system.

Here, we present a sufficient condition for the existence of robustly stabilizing switching rules for an unforced switched nonlinear system.

Theorem 1: Consider the switched nonlinear system

$$\dot{x} = f_\sigma(x) + \Delta f_\sigma(x),\ \sigma \in \{1, 2, \ldots, N\}. \tag{4}$$

If there exists a positive definite function $V: R^n \to R_0^+$ and N positive definite matrix functions $Q_1, Q_2, \ldots, Q_N: R^n \to R^{n\times n}$ satisfying the following condition:

$$\min_{i\in\{1,2,\ldots,N\}}\left\{\frac{\partial V(x)}{\partial x}f_i(x) + \frac{1}{2}\frac{\partial V(x)}{\partial x}Q_i(x)\left(\frac{\partial V(x)}{\partial x}\right)^T + \frac{1}{2}F_i^T(x)Q_i^{-1}(x)F_i(x)\right\} < 0,\ \forall x \ne 0, \tag{5}$$

then there exists a switching law σ such that the origin is the globally asymptotically stable equilibrium of system (4) for all possible uncertainties.

Proof:

Choose V as a candidate CLF for the system described by (4). For $i = 1, 2, \ldots, N$ define

$$\Phi_i \equiv \left\{x \in R^n \left| \frac{\partial V(x)}{\partial x}f_i(x) + \frac{1}{2}\frac{\partial V(x)}{\partial x}Q_i(x)\left(\frac{\partial V(x)}{\partial x}\right)^T + \frac{1}{2}F_i^T(x)Q_i^{-1}(x)F_i(x) < 0\right.\right\}.$$

The condition in (5) implies that $\bigcup_{i=1,2,\ldots,N}\Phi_i = R^n \setminus \{0\}$. Lemma 1 indicates that for each $x \in \Phi_i$, we have

$$\frac{\partial V(x)}{\partial x}\left(f_i(x) + \Delta f_i(x)\right) \le \frac{\partial V(x)}{\partial x}f_i(x) + \frac{1}{2}\frac{\partial V(x)}{\partial x}Q_i(x)\left(\frac{\partial V(x)}{\partial x}\right) + \frac{1}{2}F_i^T(x)Q_i^{-1}(x)F_i(x) < 0.$$

If the switching rule σ satisfies

$$\sigma(t) = i \text{ only if } x(t) \in \Phi_i,\ i = 1, 2, \ldots, N, \tag{6}$$

then along any nonzero trajectory of (4), we have

$$\dot{V}(x) = \frac{\partial V(x)}{\partial x}\left(f_\sigma(x) + \Delta f_\sigma(x)\right) < 0.$$

Therefore, system (4) is globally asymptotically stable under any switching law satisfying (6). This completes the proof. □

Now we consider the switched control system described by (1). We want to design a switching rule σ and a feedback controller $u = p_\sigma(x)$ such that the closed-loop system is globally asymptotically stable for all possible uncertainties.

Definition 3: A smooth positive definite and radially unbounded function $V: R^n \to R_0^+$ is a robust switched control Lyapunov function (RSCLF) of system (1) if, for each $x \ne 0$,

$$\min_{i\in\{1,2,\ldots,N\}}\max_{\Gamma_i}\inf_{u\in R^m}\left\{\frac{\partial V(x)}{\partial x}\left(f_i(x) + \Delta f_i(x) + g_i(x)u\right)\right\} < 0. \tag{7}$$

□

Definition 4: An RSCLF $V: R^n \to R_0^+$ of system (1) is said to satisfy the switched small control property (SSCP) if for each $\varepsilon > 0$ there is a $\delta > 0$ such that, if $x \ne 0$ and $\|x\| < \delta$, there is some u with $\|u\| < \varepsilon$ such that

$$\min_{i\in\{1,2,\ldots,N\}}\max_{\Gamma_i}\left\{\frac{\partial V(x)}{\partial x}\left(f_i(x) + \Delta f_i(x) + g_i(x)u\right)\right\} < 0. \tag{8}$$

□

To present the main results conveniently, for a candidate RSCLF V of system (1) let us define

$$a_i(x) = \frac{\partial V(x)}{\partial x} f_i(x) + \frac{1}{2}\frac{\partial V(x)}{\partial x} Q_i(x)\left(\frac{\partial V(x)}{\partial x}\right)^T + \frac{1}{2}F_i^T(x)Q_i^{-1}(x)F_i(x), \quad .$$

$$b_i(x) = \frac{\partial V(x)}{\partial x} g_i(x)$$

In addition, let

$$D_P = \left\{x \in R^n \,\middle|\, \min_{i\in\{1,2,\ldots,N\}} a_i(x) \ge 0\right\},$$

$$D_Z = \{x \in R^n \mid b_i(x) = 0, i = 1, 2, \ldots, N\}.$$

It can be seen that, for all possible uncertainties,

$$\frac{\partial V(x)}{\partial x}\left(f_i(x) + \Delta f_i(x) + g_i(x)u\right) \le a_i(x) + b_i(x)u, \ \forall x \in R^n. \quad (9)$$

Therefore, if for each $x \ne 0$, V is such that

$$\min_{i\in\{1,2,\ldots,N\}} \inf_{u\in R^m} \{a_i(x) + b_i(x)u\} < 0, \quad (10)$$

or equivalently,
for each $x \ne 0$ satisfying $b_i(x) = 0$, $i = 1, 2, \ldots, N$

$$\Rightarrow \min_{i\in\{1,2,\ldots,N\}} a_i(x) < 0 \quad (11)$$

then (7) holds and thus V is an RSCLF of system (1).

Therefore, either (10) or (11) is a sufficient condition for V to be an RSCLF. Note that (10) and (11) are equivalent to $D_P \cap D_Z = \{0\}$.

Lemma 2: An RSCLF $V: R^n \to R_0^+$ of system (1) satisfies the SSCP if

$$\limsup_{\varepsilon\to 0} \sup_{x\in B_\varepsilon} \min_{i\in\{1,2,\ldots,N\}} \frac{a_i(x)}{\|b_i(x)\|} \le 0. \quad (12)$$

Proof:

A $V(x)$ satisfying the SSCP is equivalent to, as $x \to 0$, finding a u with $\|u\| \to 0$ such that (8) holds. As $x \to 0$ in (12), it is clear that a u with $\|u\| \to 0$ exists such that $\min_i \{a_i(x) + b_i(x)u\} < 0$. By (9), this implies that $\min_i \left\{\frac{\partial V(x)}{\partial x}\left(f_i(x) + \Delta f_i(x) + g_i(x)u\right)\right\} < 0$ for all possible $\Delta f_i(x)$. Therefore, (8) holds and V satisfies the SSCP. □

We next present the main result.

Theorem 2: Consider the switched nonlinear control system described by (1). If there exists a smooth proper and positive definite function $V: R^n \to R_0^+$ satisfying $D_P \cap D_Z = \{0\}$ and (12), then there exists a switching rule $\sigma: [0, \infty) \to \{1, 2, \ldots, N\}$ and a feedback control law $u = p_\sigma(x)$ satisfying $\|p_\sigma(x)\| < \infty$ for $\|x\| < \infty$ and $\lim_{x\to 0}\|p_\sigma(x)\| = 0$, such that, for all possible uncertainties, the origin is the globally asymptotically stable equilibrium of this switched closed-loop system:

$$\dot{x} = f_\sigma(x) + \Delta f_\sigma(x) + g_\sigma(x)p_\sigma(x). \quad (13)$$

Proof:

For $i = 1, 2, \ldots, N$, define

$$D_{Pi} = \left\{x \in D_P \setminus \{0\} \,\middle|\, b_i(x) \ne 0 \text{ and } a_i(x)\|b_j(x)\| \le a_j(x)\|b_i(x)\| \text{ for all } j \ne i\right\},$$

$$D_{Ni} = \left\{x \in R^n \setminus D_P \,\middle|\, a_i(x) \le a_j(x) \text{ for all } j \ne i\right\},$$

$$D_{Zi} = \left\{x \in R^n \,\middle|\, b_i(x) = 0\right\},$$

and

$$D_i = D_{Pi} \cup D_{Ni}.$$

In addition, using Sontag's formula (Sontag, 1989), define

$$p_i(x) = \begin{cases} -\dfrac{a_i(x) + \sqrt{a_i^2(x) + (b_i(x)b_i^T(x))^2}}{b_i(x)b_i^T(x)} b_i^T(x), & \text{if } b_i(x) \ne 0 \text{ and } x \in D_i \\ 0, & \text{if } b_i(x) = 0 \text{ or } x \notin D_i. \end{cases} \quad (14)$$

Because $b_i(x) \ne 0$ for $x \in D_{Pi}$ and $a_i(x) < 0$ for $x \in D_{Ni}$, it can be seen that, in D_i, $a_i(x) < 0$ if $b_i(x) = 0$. Then,

$$a_i(x) + b_i(x)p_i(x) = \begin{cases} -\sqrt{a_i^2(x) + (b_i(x)b_i^T(x))^2}, & \text{if } b_i(x) \ne 0 \\ a_i(x), & \text{if } b_i(x) = 0 \end{cases} < 0, \ \forall x \in D_i. \quad (15)$$

Because $D_P \cap D_Z = \{0\}$ for each $x \in D_P \setminus \{0\}$, there is a $i \in \{1, 2, \ldots, N\}$ such that $b_i(x) \ne 0$; and for each $x \in R^n \setminus D_P$, there is a $i \in \{1, 2, \ldots, N\}$ such that $a_i(x) < 0$. It can be shown that $\bigcup_{i=1,2,\ldots,N} D_{Pi} = D_P \setminus \{0\}$, $\bigcup_{i=1,2,\ldots,N} D_{Ni} = R^n \setminus D_P$, and $\bigcup_{i=1,2,\ldots,N} D_i = R^n \setminus \{0\}$. Therefore, (15) implies that $\min_{i\in\{1,2,\ldots,N\}} \{a_i(x) + b_i(x)p_i(x)\} < 0$, $\forall x \ne 0$. Considering the condition presented in Theorem 1, with the feedback laws $p_i(x)$ defined in (14), there exists a switching rule such that the closed-loop system (13) is asymptotically stable. More precisely, if the switching rule satisfies

$$\sigma(t) = i \text{, only if } x(t) \in D_i,\ i = 1, 2, \ldots, N, \tag{16}$$

then

$$\begin{aligned}\dot{V} &= \frac{\partial V(x)}{\partial x}\left(f_\sigma(x) + \Delta f_\sigma(x) + g_\sigma(x)p_\sigma(x)\right)\\ &\le a_\sigma(x) + b_\sigma(x)p_\sigma(x)\\ &< 0,\ \ \forall x \ne 0.\end{aligned}$$

Therefore, the origin is the globally asymptotically stable equilibrium.

We next prove that, under (16) and condition (12), $\|p_\sigma(x)\| < 0$ at any bounded $x \ne 0$. By definition, the only possible points at which $p_i(x)$ is unbounded are those points in $D_i \setminus D_{Zi}$ that are extremely close to a point $\hat{x} \in \partial D_{Zi}$ (i.e., $b_i(\hat{x}) = 0$). Let $\{x^k\} \in D_i \setminus D_{Zi}$ be a sequence of states satisfying $\lim_{k\to\infty} x^k = \hat{x} \in \partial D_{Zi} \setminus \{0\}$. In D_i, we have shown that $a_i(x) < 0$ if $b_i(x) = 0$. Because $b_i(x^k) \ne 0$ and $\lim_{k\to\infty} b_i(x^k) = 0$, $\lim_{k\to\infty} a_i(x^k) < 0$. Therefore,

$$\begin{aligned}\lim_{k\to\infty}\|p_i(x^k)\| &= \lim_{k\to\infty}\left\{\frac{a_i(x^k)}{\|b_i(x^k)\|} + \sqrt{\left(\frac{a_i(x^k)}{\|b_i(x^k)\|}\right)^2 + \|b_i(x^k)\|^2}\right\}\\ &= \lim_{k\to\infty}\left\{\frac{a_i(x^k)}{\|b_i(x^k)\|} - \frac{a_i(x^k)}{\|b_i(x^k)\|}\right\} = 0.\end{aligned} \tag{17}$$

Accordingly, $p_i(x)$ is bounded for each bounded $x \ne 0$ and thus $p_\sigma(x)$ is bounded for each bounded $x \ne 0$.

Finally, we demonstrate that $\lim_{x\to 0}\|p_\sigma(x)\| = 0$. If $0 \in \bar{D}_{Ni}$, let $\{x^k\} \in D_{Ni}$ be a sequence of states satisfying $b_i(x^k) \ne 0$ and $\lim_{k\to\infty} x^k = 0$. Note that $\lim_{k\to\infty} b_i(x^k) = 0$. From the definition of D_{Ni}, we know that $a_i(x^k) < 0$ and $\lim_{k\to\infty} a_i(x^k) = 0$. Then, similar to (17), we have $\lim_{k\to\infty}\|p_i(x^k)\| = 0$.

If $0 \in \bar{D}_{Pi}$, let $\{x^k\} \in D_{Pi}$ be a sequence of states satisfying $\lim_{k\to\infty} x^k = 0$. From the definition of D_{Pi}, $\frac{a_i(x^k)}{\|b_i(x^k)\|} \le \frac{a_j(x^k)}{\|b_j(x^k)\|}$ for all $j \ne i$. Together with condition (12), we have $\lim_{k\to\infty}\frac{a_i(x^k)}{\|b_i(x^k)\|} = 0$. Because $\lim_{k\to\infty} b_i(x^k) = 0$, it is clear that

$$\lim_{k\to\infty}\|p_i(x^k)\| = \lim_{k\to\infty}\left\{\frac{a_i(x^k)}{\|b_i(x^k)\|} + \sqrt{\left(\frac{a_i(x^k)}{\|b_i(x^k)\|}\right)^2 + \|b_i(x^k)\|^2}\right\} = 0.$$

Therefore, if σ satisfies (16), $\lim_{x\to 0}\|p_\sigma(x)\| = 0$. The proof is completed. □

Note that (16) does not define an explicit switching rule. It provides only a condition for designing switching rules. By definition, $D_i \cap D_j \ne \phi$ for some $i \ne j$. The question then arises concerning whether $\sigma = i$ or $\sigma = j$ at a point $x \in D_i \cap D_j$. Similar to the hysteresis-based switching rule presented in Chapter 3 of Liberzon (2003), the following switching rule is used in this paper.

Switching rule:

Let $\sigma(0) = i$ for some i such that $x(0) \in D_i$. For each $t > 0$, if $\sigma(t^-) = i$ and $x(t) \in D_i$, keep $\sigma(t) = i$. However, if $\sigma(t^-) = i$ but $x(t) \in D_j \setminus D_i$, let $\sigma(t) = j$. It is possible that $\sigma(t^-) = i$ and $x(t) \in D_j \cap D_l \setminus D_i$ for some $j \ne l \ne i$. In this case, arbitrarily choose $\sigma(t) = j$ or $\sigma(t) = l$.

With the switching rule just described, sliding motions may occur because $D_i \cap D_j$ can define a curve or surface. This is different from the case discussed in Liberzon (2003). Nevertheless, the asymptotic stability of the closed-loop system can be guaranteed in the presence of sliding motions because V is strictly decreasing along the closed-loop trajectories on sliding surfaces.

Remark 1: The condition $D_P \cap D_Z = \{0\}$ in Theorem 2 implies that V is an RSCLF of system (1), and condition (12) implies that V satisfies the SSCP. If system (1) has an RSCLF satisfying the SSCP, then we can find a switching rule σ and feedback controller $u = p_\sigma(x)$ to stabilize it.

An RSCLF V of system (1) is not necessarily a CLF of the individual subsystems. Moreover, for the i^{th} subsystem, the feedback law $p_i(x)$ must guarantee $\dot{V} < 0$ in D_i but not in $R^n \setminus \{0\}$. Therefore, all closed-loop modes (i.e., $\dot{x} = f_i(x) + \Delta f_i(x) + g_i(x)p_i(x)$, $i = 1, 2, \ldots, N$) can be unstable. These requirements are different from those outlined in El-Farra et al. (2005). □

Remark 2: For $i = 1, 2, \ldots, N$ define

$$\hat{p}_i(x) = \begin{cases} -\dfrac{a_i(x) + \sqrt{a_i^2(x) + (b_i(x)b_i^T(x))^2}}{b_i(x)b_i^T(x)} b_i^T(x), & \text{if } b_i(x) \ne 0\\ 0, & \text{if } b_i(x) = 0.\end{cases} \tag{18}$$

For the considered problem, the intuitive choices of switching rules and feedback laws are

$$\sigma = \arg\min_{i\in\{1,2,\ldots,N\}} \{a_i(x) + b_i(x)\hat{p}_i(x)\}, \tag{19}$$

and

$$u = \hat{p}_\sigma(x). \tag{20}$$

Under the same conditions as presented in Theorem 2, the feedback law in (20) under the switching rule in (19) guarantees that $\dot{V} < 0$ along any nonzero closed-loop trajectories. This appears to be a simpler solution to the considered problem. However, given (18) and (19), the feedback law in (20) may be unbounded at some bounded x. For example, if there is some i such that $a_i(x) > \left|\sqrt{a_j^2(x) + (b_j(x)b_j^T(x))^2}\right|$ for all $j \neq i$ and $b_i(x) = 0^-$, then $a_i(x) + b_i(x)\hat{p}_i(x) = -a_i(x) < a_j(x)$ + $b_j(x)\hat{p}_j(x)$ for all $j \neq i$ and, by (19), $\sigma(x) = i$ and $\|u\|$ = $\|\hat{p}_i(x)\| \to \infty$. This controller could not be implemented. □

IV. ILLUSTRATIVE EXAMPLE

Consider the switched nonlinear control system

$$\dot{x} = f_\sigma(x) + \Delta f_\sigma(x) + g_\sigma(x)u, \ \sigma \in \{1,2\} \tag{21}$$

where

$$f_1(x) = \begin{bmatrix} -\alpha_1 x_1 - k_1 e^{-\frac{d}{x_1+c}} x_2 \\ -\alpha_1 x_2 - k_2 e^{-\frac{d}{x_1+c}} x_2 \end{bmatrix}, \ \Delta f_1(x) = \Gamma_1 \begin{bmatrix} x_1 \\ \eta_1 x_2 \end{bmatrix}$$

with

$$\|\Gamma_1\| \le 1, \ g_1(x) = \begin{bmatrix} \beta_1 \\ 0 \end{bmatrix}, \ f_2(x) = \begin{bmatrix} -\alpha_2 x_1 - k_1 e^{-\frac{d}{x_1+c}} x_2 \\ -\alpha_2 x_2 - k_2 e^{-\frac{d}{x_1+c}} x_2 \end{bmatrix},$$

$$\Delta f_2(x) = \Gamma_2 \begin{bmatrix} x_1 \\ \eta_2 x_2 \end{bmatrix} \text{ with } \|\Gamma_2\| \le 1, \ g_2(x) = \begin{bmatrix} \beta_2 \\ 0 \end{bmatrix},$$

and α_1, α_2, β_1, β_2, k_1, k_2, c, d, η_1, and η_2 are known real positive numbers. This type of dynamic equation can be used to model some chemical processes (El-Farra et al., 2005; Mhaskar et al., 2005).

Choose $V(x) = x_1^2 + \rho x_2^2$ as a candidate RSCLF for system (21). With $Q_1(x) = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$ and $Q_2(x) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}$, we have

$$\begin{aligned} a_1(x) &= \frac{\partial V(x)}{\partial x} f_1(x) + \frac{1}{2}\frac{\partial V(x)}{\partial x} Q_1(x)\left(\frac{\partial V(x)}{\partial x}\right)^T \\ &\quad + \frac{1}{2} F_1^T(x) Q_1^{-1}(x) F_1(x) \\ &= \left(-2\alpha_1 + \frac{5}{2}\right)x_1^2 + \left(-2\alpha_1\rho + \rho^2 + \eta_1^2\right)x_2^2 \\ &\quad - 2k_1 e^{-\frac{d}{x_1+c}} x_1 x_2 - 2\rho k_2 e^{-\frac{d}{x_1+c}} x_2^2, \end{aligned}$$

$$\begin{aligned} a_2(x) &= \frac{\partial V(x)}{\partial x} f_2(x) + \frac{1}{2}\frac{\partial V(x)}{\partial x} Q_2(x)\left(\frac{\partial V(x)}{\partial x}\right)^T \\ &\quad + \frac{1}{2} F_2^T(x) Q_2^{-1}(x) F_2(x) \\ &= \left(-2\alpha_2 + 2\right)x_1^2 + \left(-2\alpha_2\rho + 2\rho^2 + \frac{1}{2}\eta_2^2\right)x_2^2 \\ &\quad - 2k_1 e^{-\frac{d}{x_1+c}} x_1 x_2 - 2\rho k_2 e^{-\frac{d}{x_1+c}} x_2^2, \end{aligned}$$

$$b_1(x) = \frac{\partial V(x)}{\partial x} g_1(x) = 2\beta_1 x_1,$$

$$b_2(x) = \frac{\partial V(x)}{\partial x} g_2(x) = 2\beta_2 x_1.$$

It is clear that $D_Z = \{x \in R^2 \mid x_1 = 0\}$. That V satisfies the SSCP (i.e., that (12) holds) for any $\rho > 0$ can be easily verified. Moreover, we can show that $D_P \cap D_Z = \{0\}$ holds if $\alpha_1 > \eta_1$ and $\alpha_1 - \sqrt{\alpha_1^2 - \eta_1^2} < \rho < \alpha_1 + \sqrt{\alpha_1^2 - \eta_1^2}$, or $\alpha_2 > \eta_2$ and $\frac{\alpha_2 - \sqrt{\alpha_2^2 - \eta_2^2}}{2} < \rho < \frac{\alpha_2 + \sqrt{\alpha_2^2 - \eta_2^2}}{2}$. By Theorem 2, we can then find a switching rule σ and feedback control laws $p_1(x)$ and $p_2(x)$ such that, with $u = p_\sigma(x)$, the resultant switched closed-loop system is globally asymptotically stable.

In simulations, let $\alpha_1 = 0.4$, $\alpha_2 = 0.9$, $\beta_1 = 0.7$, $\beta_2 = 1.2$, $k_1 = 10$, $k_2 = 3$, $c = 20$, $d = 4$, $\eta_1 = 0.2$ and $\eta_1 = 0.6$. Then $\rho = 0.5$ satisfies $\frac{\alpha_2 - \sqrt{\alpha_2^2 - \eta_2^2}}{2} < \rho < \frac{\alpha_2 + \sqrt{\alpha_2^2 - \eta_2^2}}{2}$. With these parameters and using the feedback control laws constructed by (14) and the switching rule described in Section 3, the responses of the closed-loop system for arbitrary choices of Γ_1 and Γ_2 were calculated and are illustrated in Fig. 2, wherein

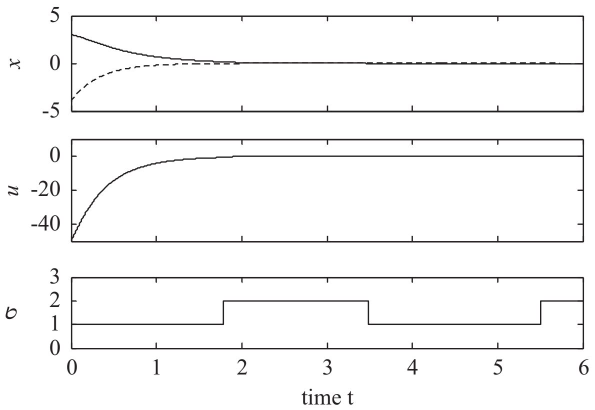

Fig. 2. Response of the closed-loop switched system.

the state trajectory is observed to asymptotically converge to the origin.

V. CONCLUSIONS

An integrated design of switching rules and feedback controllers for asymptotically stabilizing uncertain switched nonlinear control-affine systems with an arbitrary number of subsystems is discussed. An RSCLF approach was developed to derive sufficient conditions for the existence of asymptotically stabilizing switched feedback laws. The existence of RSCLFs that satisfy the SSCP was demonstrated to guarantee the existence of stabilizing switched feedback laws. An illustrative example confirmed that the proposed method is feasible for the design of switched feedback control laws for uncertain switched nonlinear control-affine systems.

ACKNOWLEDGEMENTS

This work was supported by the Ministry of Science and Technology of the Republic of China under Contract No. MOST 105-2221-E-019-031.

REFERENCES

Agrachev, A. A. and D. Liberzon (2001). Lie-algebraic stability criteria for switched systems. SIAM Journal on Control and Optimization 40, 253-269.

Artstein, Z. (1983). Stabilization with relaxed control. Nonlinear Analysis, Theory, Methods and Applications 7, 1163-1173.

Brockett, R. W. (1983). Asymptotic stability and feedback stabilization. In: Differential Geometric Control Theory. Boston, MA: Birkhauser, 181-191.

Daafouz, J., P. Riedinger and C. Iung (2002). Stability analysis and control synthesis for switched systems: A switched Lyapunov function approach. IEEE Transactions on Automatic Control 47, 1883-1887.

Dayawansa, W. P. and C. F. Martin (1999). A converse Lyapunov theorem for a class of dynamical systems which undergo switching. IEEE Transactions on Automatic Control 44, 751-760.

El-Farra, N. H., P. Mhaskar and P. D. Christofides (2005). Output feedback control of switched nonlinear systems using multiple Lyapunov functions. Systems & Control Letters 54, 1163-1182.

Hespanha, J. P. and A. S. Morse (1999). Stability of switched systems with average dwell-time. Proceeding of 38th IEEE CDC, Phoenix, AZ, 2655-2660.

Hou, M., F. Fu and G. Duan (2013). Global stabilization of switched stochastic nonlinear systems in strict-feedback form under arbitrary switching. Automatica 49, 2571-2575.

Krstic, M., I. Kanellakopoulos and P. Kokotovic (1995). Nonlinear and Adaptive Control Design. New York: Interscience.

Liberzon, D. (2003). Switching in Systems and Control. Boston, MA: Birkhauser.

Liberzon, D., J. P. Hespanha and A. S. Morse (1999). Stability of switched systems: A Lie-algebraic condition. Systems & Control Letters 37, 117-122.

Liberzon, D. and A. S. Morse (1999). Basic problems in stability and design of switched systems. IEEE Control Syst. Mag. 19, 59-70.

Lin, H. and P. J. Antsaklis (2009). Stability and stabilizability of switched linear systems: a survey of recent results. IEEE Transactions on Automatic Control 54, 308-322.

Long, L. and J. Zhao (2011a). Global stabilization for a class of switched nonlinear feedforward systems. Systems & Control Letters 60, 734-738.

Long, L. and J. Zhao (2011b). Global stabilization of switched nonlinear systems in p-normal form with mixed odd and even powers. International Journal of Control 84, 1612-1626.

Ma, R. and J. Zhao (2010). Backstepping design for global stabilization of switched nonlinear systems in lower triangular form under arbitrary switchings. Automatica 46, 1819-1823.

Mhaskar, P., N. H. El-Farra and P. D. Christofides (2005). Predictive control of switched nonlinear systems with scheduled mode transitions. IEEE Transactions on Automatic Control 50, 1670-1680.

Petersen, I. R. (1987). A stabilization algorithm for a class of uncertain linear systems. Systems & Control Letters 8, 351-357.

Sontag, E. D. (1983). A Lyapunov-like characterization of asymptotic controllability. SIAM J. Control & Optimization 21, 462-471.

Sontag, E. D. (1989). A 'universal' constructive of Artstein's theorem on nonlinear stabilization. Systems & Control Letters 12, 542-550.

Wang, J. Q. and C. F. Yung (2016). Robust Stabilization of Uncertain Switched Nonlinear Systems with Two Modes. To be published in Journal of Marine Science and Technology, 2016.

Wu, J. L. (2008). Feedback Stabilization for Multi-Input Switched Nonlinear Systems: Two Subsystems Case. IEEE Transactions on Automatic Control 53, 1037-1042.

Wu, J. L. (2009). Stabilizing Controllers Design for Switched Nonlinear Systems in Strict-Feedback Form. Automatica 45, 1092-1096.

Xu, X. and P. J. Antsaklis (2004). Optimal control of switched systems based on parameterization of the switching instants. IEEE Transactions on Automatic Control 49, 2-16.

Yang, H., V. Cocquempot and B. Jiang (2009). On stabilization of switched nonlinear systems with unstable modes. Systems & Control Letters 58, 703-708.

Yang, H., B. Jiang, V. Cocquempot and H. Zhang (2011). Stabilization of switched nonlinear systems with all unstable modes: application to multi-agent systems. IEEE Transactions on Automatic Control 56, 2230-2235.

Ye, H., A. N. Michel and L. Hou (1998). Stability theory for hybrid dynamical systems. IEEE Transactions on Automatic Control 43, 461-474.

Zefran, M. and J. W. Burdick (1998). Design of switching controllers for systems with changing dynamics. Proceeding of 37th IEEE CDC, 2113-2118.

Zhao, J. and G. M. Dimirovski (2004). Quadratic stability of a class of switched nonlinear systems. IEEE Transactions on Automatic Control 49, 574-578.

Zhao, J. and D. J. Hill (2008). On stability, L_2-gain and H_∞ control for switched systems. Automatica 44, 1220-1232.