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A NUMERICAL IDENTIFICATION OF EXCITATION FORCE AND NONLINEAR RESTORING CHARACTERISTICS OF SHIP ROLL MOTION

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Key words: ship roll motion, excitation force, nonlinear restoring characteristics, *J*-function, zero-crossing time.

ABSTRACT

The present work identifies the nonlinear restoring characteristics and excitation force of ship roll motion in regular beam waves. The steady-state roll response data measured from a numerical simulation is utilized for the experiments of the applied method. Using the approach proposed by Jang (2014), the *J*-function and its one of zero-crossing times play a crucial role in the identification. The *J*-function is defined to be the sum of inertia force and nonlinear damping. For the zero-crossing time to be measured, system responses of angular displacement, velocity and acceleration need to be known and so does the numerical interpolation. Numerical simulations using the ship models are conducted to show the workability and validity of the methodology. These numerical simulations show that Jang's scheme is an effective and very simple method for the identification of the characteristics of ship roll motion and produces relatively reasonable solutions.

I. INTRODUCTION

In general, ships experience six degrees of freedom motion: surge, sway, heave, roll, pitch, and yaw, while the most critical mode of motion causing a ship to capsize is the roll motion. For most of the time, roll motion of considerable magnitude is generated by beam sea waves, and sometimes following and quartering sea waves also may induce roll motion. It is also well known that roll motion tends to exhibit strong resonant motion. So the stability analysis against capsizing in rolling motion is one of the critical steps of a ship design. Thus there is much excellent research on the assessment of ship's stability considering the nonlinear restoring force caused by the external excitation force.

There are numerous studies on the subject of investigating the restoring characteristics of a ship; Bhattacharyya (1978) expressed the nonlinear uncoupled rolling motion of a ship using for the restoring moment odd-polynomials comprising linear, cubic and quintic terms. Cardo et al. (1984), Nayfeh and Khdeir (1986) and Senjanović (1994) investigated the effects of nonlinear restoring characteristics of a ship roll motion. Taylan (1999, 2000, 2003, 2007) developed the mathematical modeling of the nonlinear roll motion and parametric resonance. He also studied a rolling ship in regular beam waves using various representations of nonlinear damping and nonlinear restoring terms. Surendran and Reddy (2003) assessed the ship stability for various dynamic environments considering three types of nonlinear damping and a quintic model for the restoring force. Recently, Peşman and Taylan (2012) examined the influence of rolling ship's restoring moment curve on the parametric roll motion in regular longitudinal waves. And there's also general method for the simultaneous identification both of nonlinear restoring and nonlinear damping characteristics (Peşman and Taylan, 2012).

Concerning the excitation force identification, there are also numerous excellent pieces of research. To name a few, Faltinsen (1990) analyzed diverse sea loads on ships and offshore structures such as wind, waves and tried to demonstrate how numerical methods could be utilized in practice for those engineering problems. Pessoa et al. (2011) investigated the first and second order wave exciting forces acting on a ship of simple geometry in long crested irregular waves. Hirdaris et al. (2014) reviewed the literature of loads identification for the design of ships and offshore structures for the environmental and operational loads such as waves, wind, current, ice, slamming, sloshing and operational factors from model experiments, full-scale measurements and theoretical methods.

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As shown above, there is much excellent research on the identification either of nonlinear restoring characteristic or external excitation force of a ship. However, there are few works on the simultaneous identification of both of them. Penna et al. (1997) discussed the uncertainty analysis applied to the parameter estimation in nonlinear rolling. Francescutto et al. (1998) studied the effect of excitation modelling in the parameter estimation regards the excitation parameters. Recently, Jang (2014) proposed a new theory in the form of an inverse problem for the simultaneous identification of the restoring and the excitation forces of a general nonlinear forced system. For this purpose, he developed the new notion of the J-function and its zero-crossing time; he also mathematically showed the uniqueness and the stability of the solution. Jang and Park (2014) numerically investigated the linear restoring force and the external harmonic excitation force in a general linear oscillating system. Ahmad et al. (2016) detected the nonlinear restoring and the excitation forces using an integral scheme. Interestingly enough, this integral scheme utilized by Ahmad et al. (2016) detected the nonlinear restoring and the excitation terms simultaneously.

In the present study, based on the method proposed by Jang (2014), both the nonlinear restoring characteristics and the external excitation force of a rolling ship in regular beam sea waves are investigated. For that, the measured system responses and the zero-crossing times measured from both of *J*-function its derivatives with respect to time, *dJ*/*dt* (Jang, 2014; Jang and Park, 2014) are used. For the numerical simulations, the applied model ships and their properties are provided from the conventional articles (Taylan, 1999; Gu, 2004). The nonlinear damping characteristics (quadratic and cubic model which will be shown in the next section) are assumed to be known for the present simulations, although estimating damping is extremely difficult. The representation of the unknown nonlinear restoring force is limited to the linear-plus-cubic model. Using the assumptions, two numerical simulations are conducted for simultaneous identification both of the nonlinear restoring characteristics and the excitation force. As a conclusion, the present methodology is found to be simple for the parametric identification and produces relatively reasonable solutions for the recovering a ship's roll motion.

II. EQUATION OF ROLL MOTION

A typical ship roll motion in regular beam sea waves is expressed as:

$$I\ddot{\phi} + B(\dot{\phi}) + C(\phi) = M(t), \tag{1}$$

where ϕ is the roll angle (or angular displacement in radian), I is the sum of mass moment of inertia and the added mass moment of inertia, $B(\dot{\phi})$ is the nonlinear damping characteristic dependent on the angular velocity, $C(\phi)$ is the nonlinear restoring force and $M(t)$ is the external force term.

In the present study, only the roll motion which is the significant motion pertaining to stability and capsizing has been considered. A ship is assumed to be subjected to a regular sinusoidal beam sea wave, and the roll motion is assumed uncoupled from all others. The added mass moment of inertia is assumed to be independent of the excitation frequency; this means that the total mass moment of inertia is constant. The external excitation force is harmonic with the frequency of waves. Although there exist much researches on frequency-domain solutions, a time-domain solution procedure is here utilized to apply the *J*-function (defined in the next section) and measure one of its zero-crossing times.

The nonlinear damping moment term $B(\dot{\phi})$ in Eq. (1) is represented by two cases of damping models (B_{LQ} and B_{LC}):

$$B_{LQ}(\dot{\phi}) = B_{L,1}\dot{\phi} + B_Q\dot{\phi}\left|\dot{\phi}\right| \tag{2}$$

$$B_{LC}(\dot{\phi}) = B_{L,2}\dot{\phi} + B_C\dot{\phi}^3 \tag{3}$$

Taylan (1999, 2003), Surendran and Reddy (2003) and Contento et al. (1996) studied the linear-plus-quadratic damping type of Eq. (2). Dalzell (1978), Francescutto and Contento (1999) and Contento et al. (1999) studied the linear-plus-quadratic model and the linear-plus-cubic damping one for an ideal ship in nonlinear rolling. The main difference of the two damping ones results from the saturation phenomenon at peak and the cubic one depends on the hull and on the appendages (Francescutto and Contento, 1999).

III. DETERMINATION OF NONLINEAR RESTORING AND EXCITATION FORCE

The *J*-function, which is the summation of inertia and nonlinear damping terms, is defined as (Jang, 2014)

$$J(\dot{\phi}, \ddot{\phi}) = I\ddot{\phi} + B(\dot{\phi}). \tag{4}$$

Thus, the system responses of angular displacement, velocity and acceleration should be measured or estimated prior to its identification. For the mathematical development of the present methodology, its derivative with respect to time, *dJ*/*dt*, is also required.

Generally the angular velocity could be easily measured via gyro sensor. However, the measurement or the estimation of angular displacement and acceleration should be done with caution due to gravitational effect and drift characteristics of gyro sensors. If the given body object moves within a particular bounded region, then non-contact optical motion measurement sensors such as RODYM could be utilized in practice to measure angular displacement with sufficient accuracy. Subsequently, by using a numerical derivative method or difference scheme, one can estimate the angular velocity and angular acceleration, which is in fact carried out in the present study.

In Eq. (4), the angular velocity $\dot{\phi}$ can be obtained by using

4^{th} order Runge-Kutta scheme and the angular acceleration $\ddot{\phi}$ does by using numerical differentiation one such as five-point stencil in one dimension. So that *J*-function in Eq. (4) can be obtained. And to calculate *dJ*/*dt*, the same numerical derivative of *J*-function with respect to time using five-point stencil is also utilized. The zero-cross times of *J* and *dJ*/*dt* are designated as t_J^{ze}, $t_{dJ/dt}^{ze}$ (> 0), respectively (Jang, 2014).

In the present study, the unknown nonlinear restoring force in Eq. (1) is approximated as having the following cubic form for simplicity (Taylan, 1999)

$$C(\phi) = k_1\phi + k_3\phi^3, \tag{5}$$

where k_1 and k_3 are the linear and nonlinear restoring force coefficients, respectively. In practice, it is usual to use the cubic approximation in nonlinear modeling of restoring forces of a ship's roll motion. The external harmonic excitation force is defined as $M(t) \equiv \gamma \cdot \cos(\omega t)$. With the use of an idea originally proposed by Jang (2014), coefficients k_1 and k_3 as well as the amplitude γ are analytically expressed as

$$\gamma = \lim_{t_0 \to \infty} \frac{\int_{t_0}^{T+t_0} B(\dot{\phi}) \cdot \dot{\phi}\, dt}{\int_{t_0}^{T+t_0} (\cos \omega t) \cdot \dot{\phi}\, dt}, \tag{6}$$

$$k_1 = \gamma \cdot \frac{-3\phi^2(t_{dJ/dt}^{ze}) \cdot \dot{\phi}(t_{dJ/dt}^{ze}) \cdot \cos(\omega t_J^{ze}) - \omega \cdot \phi^3(t_J^{ze}) \cdot \sin(\omega t_{dJ/dt}^{ze})}{-3\phi^2(t_{dJ/dt}^{ze}) \cdot \dot{\phi}(t_{dJ/dt}^{ze}) \cdot \phi(t_J^{ze}) + \dot{\phi}(t_{dJ/dt}^{ze}) \cdot \phi^3(t_J^{ze})}, \tag{7}$$

$$k_3 = \gamma \cdot \frac{\dot{\phi}\left(t_{dJ/dt}^{ze}\right) \cdot \cos\left(\omega t_J^{ze}\right) + \omega \cdot \phi\left(t_J^{ze}\right) \cdot \sin\left(\omega t_{dJ/dt}^{ze}\right)}{-3\phi^2\left(t_{dJ/dt}^{ze}\right) \cdot \dot{\phi}\left(t_{dJ/dt}^{ze}\right) \cdot \phi\left(t_J^{ze}\right) + \dot{\phi}\left(t_{dJ/dt}^{ze}\right) \cdot \phi^3\left(t_J^{ze}\right)}. \tag{8}$$

where t_0 is a sufficiently long time when the roll motion balances an external, or steady-state excitation with period *T*.

For the more accurate calculation of the coefficients k_1 and k_3, simple linear interpolating will decrease the numerical errors. The coefficients k_1 and k_3 are given with the linearly interpolated parameters, (see, Appendix A)

$$k_1 = \gamma \cdot \frac{-3\phi^2(t_{dJ/dt}^{ze,LI}) \cdot \dot{\phi}(t_{dJ/dt}^{ze,LI}) \cdot \cos\left(\omega\, t_J^{ze,LI}\right) - \omega \cdot \phi^3(t_J^{ze,LI}) \cdot \sin\left(\omega\, t_{dJ/dt}^{ze,LI}\right)}{-3\phi^2\left(t_{dJ/dt}^{ze,LI}\right) \cdot \dot{\phi}\left(t_{dJ/dt}^{ze,LI}\right) \cdot \phi\left(t_J^{ze,LI}\right) + \dot{\phi}\left(t_{dJ/dt}^{ze,LI}\right) \cdot \phi^3\left(t_J^{ze,LI}\right)}, \tag{9}$$

$$k_3 = \gamma \cdot \frac{\dot{\phi}\left(t_{dJ/dt}^{ze,LI}\right) \cdot \cos\left(\omega\, t_J^{ze,LI}\right) + \omega \cdot \phi\left(t_J^{ze,LI}\right) \cdot \sin\left(\omega\, t_{dJ/dt}^{ze,LI}\right)}{-3\phi^2\left(t_{dJ/dt}^{ze,LI}\right) \cdot \dot{\phi}\left(t_{dJ/dt}^{ze,LI}\right) \cdot \phi\left(t_J^{ze,LI}\right) + \dot{\phi}\left(t_{dJ/dt}^{ze,LI}\right) \cdot \phi^3\left(t_J^{ze,LI}\right)}. \tag{10}$$

IV. NUMERICAL SIMULATIONS

The roll motion responses are solved numerically using the 4^{th} order Runge-Kutta integration scheme and the Simpson's 3/8 rule for the numerical integration and the five-point stencil in one dimension for the numerical differentiation.

Table 1. Principal properties of the ship (Taylan, 1999).

Length between perpendiculars	L_{BP} [*m*]	56.78
Breadth	*B* [m]	12.20
Depth	*D* [m]	6.80
Draft (mean)	T_M [m]	4.10
Trim (by stern)	- [m]	1.20
Block coefficient	C_B [-]	0.515
Displacement	Δ [ton]	1,567

Table 2. Coefficients of the Eq. (11) (Taylan, 1999).

$I_{xx} + \delta I_{xx}$	[kg · m · sec^2]	63,555.0
$B_{L,1}$	[kg · m · sec]	6,172.0
B_Q	[kg · m · sec^2]	10,735.0
$k_{1,1}$	[kg · m]	10,454.0
$k_{3,1}$	[kg · m]	1,316.84
α_m	[rad]	0.2
ω	[rad · sec^{-1}]	0.407
γ	[kg · m]	1,684.5

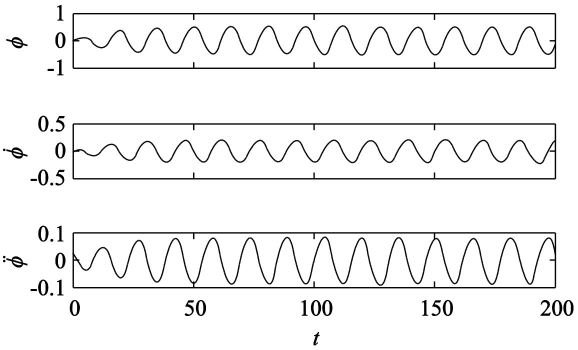

Fig. 1. System responses obtained by solving Eq. (11) using the properties of Table 2 and the zero initial conditions.

1. System Responses

Case 1

Introducing in Eq. (1), the governing nonlinear roll motion, the nonlinear restoring force given by Eq. (5) and the nonlinear damping model given by Eq. (2) with the external harmonic excitation force gives (Taylan, 1999),

$$\begin{aligned}\left(I_{xx} + \delta I_{xx}\right)\ddot{\phi} + B_{L,1}\dot{\phi} + B_Q\dot{\phi}\left|\dot{\phi}\right| + k_{1,1}\phi + k_{3,1}\phi^3 \\ = \omega^2 \alpha_m I_{xx} \cdot \cos\omega\, t = \gamma \cdot \cos\omega\, t.\end{aligned} \tag{11}$$

In the present simulation, the added mass moment of inertia is set to be 25% of the mass moment of inertia. The principal properties of the model ship and the parameters in Eq. (11) are listed in Table 1 and Table 2. Fig. 1 shows the angular dis-

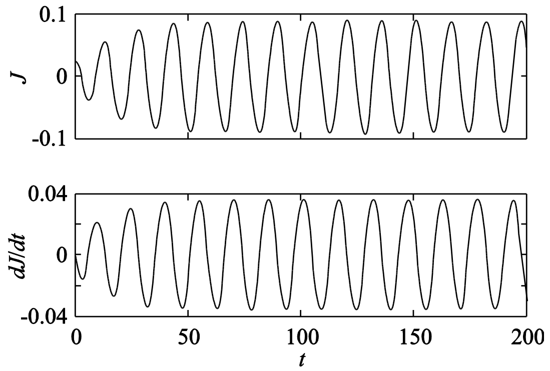

Fig. 2. *J*-function and its derivative with respect to time: case 1.

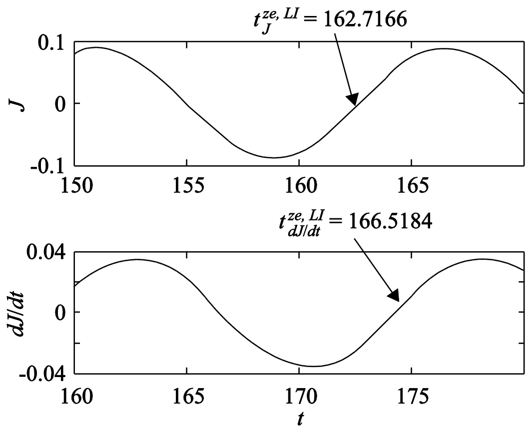

Fig. 3. The measured zero-crossing times $t_J^{ze,LI}$ and $t_{dJ/dt}^{ze,LI}$.

placement, velocity and acceleration with zero initial condition of a ship in regular sinusoidal waves or beam sea waves (t = [0, 200], Δt = 0.01). Here, the angular acceleration is numerically calculated from the velocity data. Fig. 2 depicts the J-function and its derivative with respect to time, or, dJ/dt.

The system responses shown in Fig. 1 reach steady state of a limit cycle with period T after a sufficiently long time. In a limit cycle, the work done by the damping force is balanced to the external harmonic load. So the harmonic load amplitude, γ, can be calculated using Eq. (6).

Using the Eqs. (9) and (10), the linear and the nonlinear restoring coefficients are easily obtained just by estimating (or interpolating) the zero-crossing times $t_J^{ze,LI}$ and $t_{dJ/dt}^{ze,LI}$, which are randomly chosen from steady state response data with period T after a sufficiently long time of steady-state. The plots in Fig. 3 are enlarged versions of those in Fig. 2 demonstrating the values of the linearly interpolated zero-crossing times designated as $t_J^{ze,LI}$ and $t_{dJ/dt}^{ze,LI}$.

Case 2

$$(I_{xx}+\delta I_{xx})\ddot{\phi}+B_{L,2}\dot{\phi}+B_C\dot{\phi}^3+k_{1,2}\phi+k_{3,2}\phi^3=\gamma\cdot\cos\omega t\,, \quad (12)$$

Table 3. Principal properties of the ship (Gu, 2004).

Length overall	L_{OA} [m]	30.70
Length between perpendiculars	L_{BP} [m]	25.00
Breadth	B [m]	6.90
Depth	D [m]	4.96
Draft (mean)	T_M [m]	2.67
Displacement	Δ [ton]	195

Table 4. Coefficients of the Eq. (12) (Gu, 2004).

$I_{xx}+\delta I_{xx}$	[kg · m · sec^2]	1.078×10^6
$B_{L,2}$	[kg · m · sec]	22,420
B_C	[kg · m · sec3]	17,770
$k_{1,2}$	[kg · m]	187,590
$k_{3,2}$	[kg · m]	42,510
ω	[rad · sec-1]	0.4
γ	[kg · m]	10,780

Table 5. Results compared to the exact parameters (Taylan, 1999, Gu, 2004).

	Case 1				Case 2		
	Exact	Numerical	*Err* (%)		Exact	Numerical	*Err* (%)
γ	1,684.5	1,684.2	0.0144	γ	10,780	10,792	0.1107
$k_{1,1}$	10,454.0	10,470.0	0.1544	$k_{1,2}$	187,590	187,710	0.0637
$k_{3,1}$	1,316.8	1,314.9	0.1449	$k_{3,2}$	42,510	42,780	0.6341

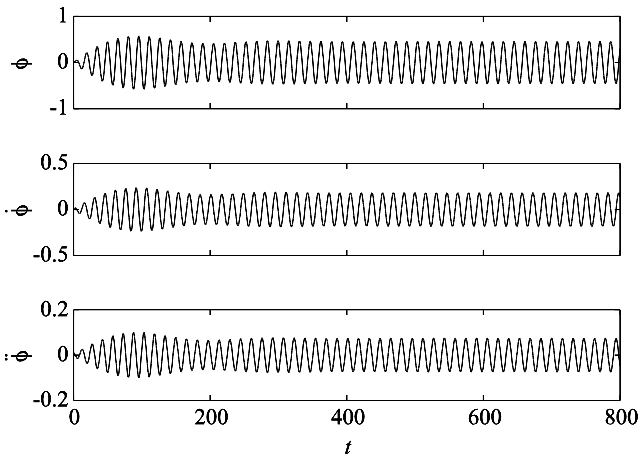

Fig. 4. System responses of Eq. (12) with zero initial conditions: case 2.

where the values of the coefficients are as indicated in Tables 3 and 4 (Gu, 2004). Fig. 4 demonstrates the system responses obtained as described in case 1 (t = [0, 800], Δt = 0.01) with zero initial condition and Figs. 5 and 6 show the calculated J-function, its time derivative, dJ/dt and their enlarged version with linearly interpolated zero-crossing times, respectively.

V. RESULTS AND DISCUSSION

Table 5 allows a comparison between the calculated external

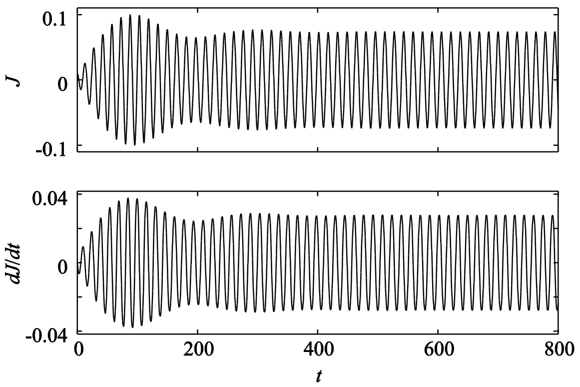

Fig. 5. *J*-function and its derivative with respect to time: case 2.

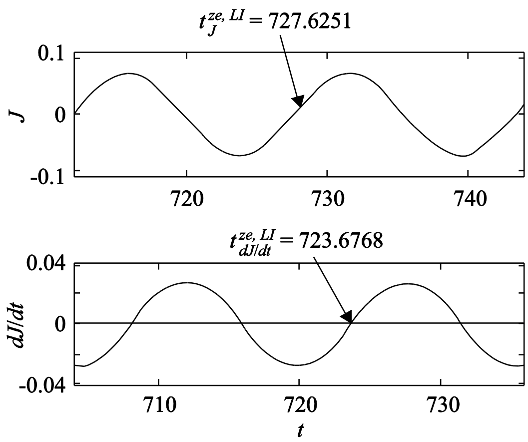

Fig. 6. The measured zero-crossing times $t_J^{ze,LI}$ and $t_{dJ/dt}^{ze,LI}$.

excitation force amplitude γ, the coefficient of linear restoring force k_1 and the nonlinear one k_3 and their respective exact values (Taylan, 1999, Gu, 2004). Fig. 7 shows the recovered restoring forces for each case over the expanded range, [-1.2, 1.2] while the actual roll angle domain is [-0.5, 0.5] in the given cases. The two plots in Fig. 8 demonstrate the recovered system responses using the numerically obtained parameters listed in Table 5 compared to those of exact one by using 4th order Runge-Kutta integration scheme using the error definition

$$Err(\%) = \frac{\sqrt{\sum_{i=1}^{N}\{\varphi_{exact}(t_i) - \varphi_{numerical}(t_i)\}^2}}{\sqrt{\sum_{i=1}^{N}\{\varphi_{exact}(t_i)\}^2}} \times 100\,, \qquad (13)$$

and the errors of the recovered responses compared to the exact one in the plots of Fig. 8 are Err = 0.4502% for case 1 and Err = 0.4657% for case 2, respectively.

VI. CONCLUSIONS

The present work applies a new method proposed by Jang (2014) for the simultaneous identification of nonlinear restoring force characteristics and the harmonic excitation force of the roll motion of a ship in regular beam wave. For this purpose, angular displacement and angular velocity need to be measured and the acceleration calculated using the high order numerical

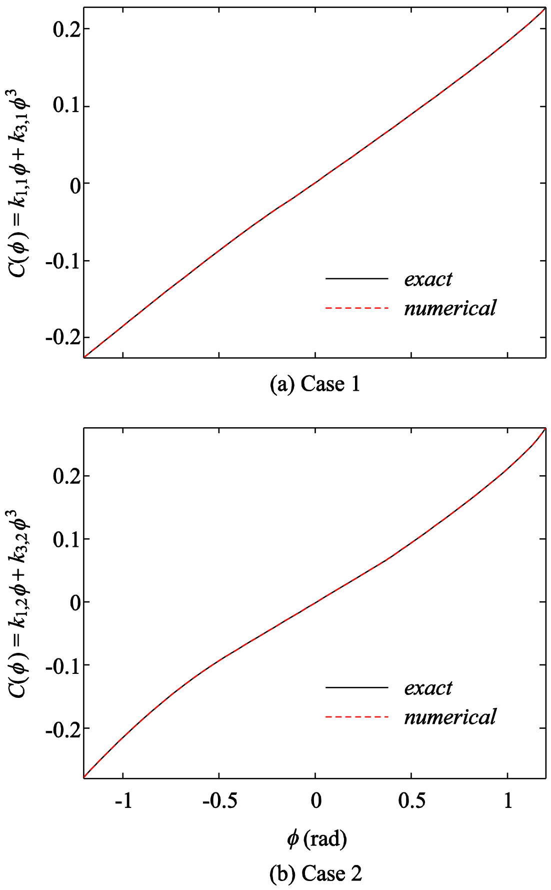

Fig. 7. Recovered nonlinear restoring characteristics compared to exact ones.

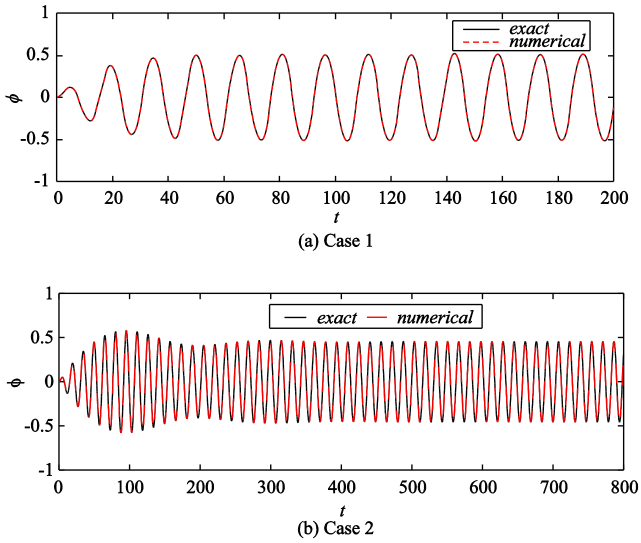

Fig. 8. Recovered system response compared to the exact one.

differentiation. The present work assumes the nonlinear damping characteristics are known. The sum of moment of inertia and the nonlinear damping terms, or *J*-function is obtained and its zero-crossing times, which play an important role on the identification, are measured or numerically interpolated. Numerical simulations are conducted with 2 cases of roll motion model for identification both of their nonlinear restoring characteristics and external excitation force. It is thus shown that Jang's scheme can be applied to the ship roll motion, it is very simple, and provides relatively reasonable numerical solutions. For the future works, the effectiveness of the method will be tested with real ship motion data.

VII. NOMENCLATURE

ϕ	Roll angle or angular displacement in radian
$\dot{\phi}$	Roll angular velocity
$\ddot{\phi}$	Roll angular acceleration
I	Moment of inertia
I_{xx}	Mass moment of inertia
δI_{xx}	Added mass moment of inertia
$B(\dot{\phi})$	Nonlinear damping moment term
$C(\phi)$	Nonlinear restoring moment term
$M(t)$	Exciting force of external waves
$J(\dot{\phi}, \ddot{\phi})$	*J*-function
dJ/dt	Derivative of *J* with respect to time
k_1, $k_{1,1}$, $k_{1,2}$	Linear restoring coefficient
k_3, $k_{3,1}$, $k_{3,2}$	Cubic nonlinear restoring coefficient
γ	External excitation force coefficient
ω	Frequency
t_0	A sufficiently long time instant after reaching steady-state oscillation in solution
T	Time period in steady-state oscillation
α_m	Effective slope of the sea state
B_L, $B_{L,1}$, $B_{L,2}$	Linear damping coefficient
B_Q	Quadratic nonlinear damping coefficient
B_C	Cubic nonlinear damping coefficient
Δ	Displacement of ship hull
$t_J^{ze,LI}$	Linearly interpolated zero-crossing time of *J*-function
$t_{dJ/dt}^{ze,LI}$	Linearly interpolated zero-crossing time of *dJ*/*dt*

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APPENDIX A. LINEAR INTERPOLATION FOR EQS. (9) AND (10)

The zero-crossing times t_J^{ze} and $t_{dJ/dt}^{ze}$ can be linearly interpolated using two sets of data pairs: $\left(t_a, J(t_a)\right)$ and $\left(t_b, J(t_b)\right)$, $\left(t_c, dJ/dt(t_c)\right)$ and $\left(t_d, dJ/dt(t_d)\right)$. Those linearly interpolated zero-crossing times are noted as $t_J^{ze,LI}$ and $t_{dJ/dt}^{ze,LI}$ (Jang and Park, 2014):

$$t_J^{ze,LI} = t_b - J(t_b) \cdot \frac{\Delta t}{J(t_b) - J(t_a)}, \tag{A.1}$$

$$t_{dJ/dt}^{ze,LI} = t_d - dJ/dt(t_d) \cdot \frac{\Delta t}{dJ/dt(t_d) - dJ/dt(t_c)}. \tag{A.2}$$

$\phi(t_J^{ze})$, $\phi(t_{dJ/dt}^{ze})$ and $\dot{\phi}(t_{dJ/dt}^{ze})$ in Eqs. (7) and (8) can be also linearly interpolated by using the zero-crossing times in Eqs. (A.1) and (A.2) (Jang and Park, 2014):

$$\phi(t_J^{ze,LI}) = \phi(t_b) - (t_b - t_J^{ze}) \frac{\phi(t_b) - \phi(t_a)}{\Delta t}, \tag{A.3}$$

$$\phi(t_J^{ze,LI}) = \phi(t_d) - (t_d - t_{dJ/dt}^{ze}) \frac{\phi(t_d) - \phi(t_c)}{\Delta t} \tag{A.4}$$

and

$$\dot{\phi}(t_{dJ/dt}^{ze,LI}) = \dot{\phi}(t_d) - (t_d - t_{dJ/dt}^{ze}) \frac{\dot{\phi}(t_d) - \dot{\phi}(t_c)}{\Delta t}, \tag{A.5}$$

respectively.

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