

A SIMPLIFIED ANALYTICAL METHOD FOR ESTIMATING FLOW-INDUCED VIBRATION RESPONSES OF HYDRAULIC STRUCTURES CAUSED BY FLOW FLUCTUATION

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Key words: flow fluctuation, hydraulic structure, flow-induced vibration, equilibrium equation, westergaard.

ABSTRACT

In many hydraulic engineering investigations of flow-induced vibration responses, a model test is necessary but yet expensive. In this article, an equilibrium equation between the operation of fluctuating pressure and the work done by damping is established to provide a simplified analytical method for estimating the flow-induced vibration responses of hydraulic structures caused by water flow fluctuations. This approach is based on the Westergaard solution. The flow-induced vibration of water flowing around a dividing wall, the flood discharge vibration of the Liuxihe arch dam, and the flow-induced vibration of the Ertan hyperbolic arch dam are presented to explain the detailed procedure of this method. In addition, three examples are given to prove the feasibility of this technique. Among these, the calculated vibration value of the Ertan hyperbolic arch dam is slightly greater than those obtained from the results of the hydro-elastic model that was based on experimental and prototype observations. If the calculated value of the vibration is relatively small based on the proposed method, carrying out model tests therefore becomes irrelevant. The results presented in this paper are useful and applicable to hydraulic engineering.

I. INTRODUCTION

Flow-induced vibration is a typical fluid-solid coupling phenomenon. The structural vibration is subjected to the random hydrodynamic load induced by water flow, while the random vibration of the structure is caused by the disturbance of the flow

field. These two phenomena interact and also establish a feedback relationship to form a certain state of the water-solid system motion (Naudascher and Rockwell, 2012). The study of the motion of the system can be categorically placed under the hydro-elastic discipline. This paper discusses only the hydraulic structure's flow-induced vibration caused by the flow fluctuation.

The primary research methods of flow-induced vibration are as follows: numerical simulation method, physical model test method, combination method of physical model test, and numerical simulation. So far, it has been difficult to solve the basic equations of the motion of flow-induced vibration system. The complexity of the overall calculation is not the structural calculation aspect, but the load duration acting on the structure. Although, in principle, the fluctuating pressure can be obtained via the numerical solution of the N-S and Reynolds equations based on Large Eddy Simulation (Tian et al., 2014; Cao and Tamura, 2016; Launchbury, 2016; Leonard, 2016). However, the reliability of this method is not very effective for the complicated geometric boundary.

In the field of civil engineering, the coupling between the elastic and water bodies originates from the coupling of the dam and water. In numerical analysis, the additional mass method of water can be taken into account. Presently, some commercial structure calculation programs can obtain additional mass by approximate calculation. The disturbance of structural vibration on the fluid field as a result of a series of sources in the fluid boundary allows the relationship between the perturbation pressure and the disturbance acceleration to be determined based on the analysis of the relationship between the source intensity and the disturbance pressure. Following this approach, the additional mass matrix is then determined. Aydin and Demirel (2012) developed a finite volume model applied to a dam-reservoir system to analyze the hydrodynamic behavior of reservoir water during earthquakes. The mathematical model is based on the solution of two-dimensional (2D) Navier-Stokes equations in a vertical semi-infinite domain truncated by a far-end boundary condition. Wang et al. (2013) analyzed the nonlinear earthquake response of arch dams using a comprehensive model. The following factors were taken into account: the semi-unbounded size

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of foundation rock and compressible water, the opening of contraction joints, the cracking of the dam body, and the spatial variation of ground motions. Gupta et al. (2017) presented a response spectrum-based stochastic method for obtaining statistical estimates of various response quantities of gravity dams, in which reservoir interactions were modeled realistically using a substructure approach. Mircevska et al. (2017) presented a theoretical development of a practical numerical method for the analysis of complex dam-reservoir interaction problems. The approach took advantage of the combination of boundary and finite element methods for the independent analysis of two physically distinct dam structure and reservoir-foundation domains. However, these numerical methods neglect the effect of additional damping and stiffness of the water body. At the same time, the complexity of the boundary conditions is caused by the intricate nature of the hydraulic and marine structures. The boundary condition of the structure is set up correctly in the case of the numerical simulation technique so that it is consistent with the actual running state, which will determine the correctness of the final calculation results.

Because the flow-induced vibration problem is quite complicated, the numerical simulation method is thus limited. Therefore, it is also necessary to examine the internal mechanism of flow-induced vibration and the effect of convection-excited vibration under operation conditions depending on the experimental research. Model tests usually employ a hydro-elastic approach, i.e., the model satisfies both the gravity and Cauchy similarities. During the early development period of this research area, incomplete elastic material for the model was widely used globally. This material was also referred to as the abnormal hydro-elastic model. The model has large error and is incapable of measuring the dynamic stress of the model. The introduction of hydro-elasticity material addresses this problem. This particular model which is made of water elastic material is called the full water elastic model (Haszpra, 1979; Jiefang, et al., 1999). The combination of the physical model test and numerical simulation is so as to measure the fluctuating pressure only on a gravity similarity model. The prototype is then changed and the structural vibration of this new variation is determined through numerical calculation. Khot et al. (2017) investigated the effect of various parameters such as pipe diameter, pipe wall thickness, and volumetric flow rate (discharge) on the amplitude of pipe vibration in straight pipe for turbulent flow via experimental and simulation techniques. Tian et al. (2017) studied the flow-induced vibration of propeller blades under periodic inflows based on experimental and simulation methods.

However, these existing techniques are all labor intensive. Therefore, this article proposes a simple quantitative method for estimating flow-induced vibration. If the calculated value of the vibration is relatively small based on the proposed method, conducting model tests becomes insignificant. This article addresses only the hydraulic structure's flow-induced vibration caused by the flow fluctuation. For illustration purposes, three numerical examples were used to demonstrate this method.

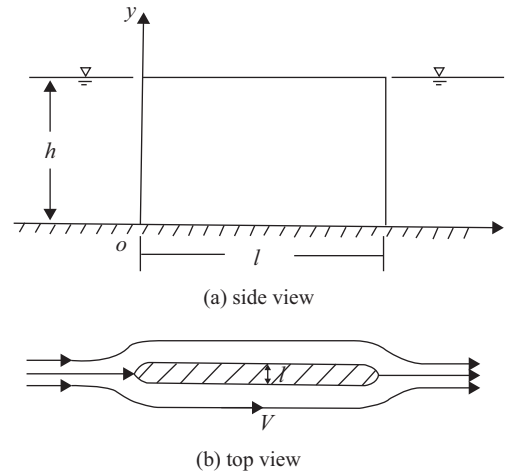


Fig. 1. Diagram of water flow around a partition.

II. FLOW-INDUCED VIBRATION OF WATER FLOWING AROUND A PARTITION

Because the water pressure pulsation frequency is lower under normal circumstances, the vibration response in a system with multiple degrees of freedom of the first structure is evaluated. According to this approach, the vibration amplitude of the convection can be simply estimated by approximation. A diagrammatic representation of water flow around a partition is shown in Fig. 1, where the depth of h is equal to the wall height, wall length is l , and velocity is V .

Assuming that the effect of the partition on the turbulent fluctuation pressure for harmonic force is $p(t)$ along the whole structure and is uniformly distributed, the following formula is derived,

$$p(t) = p_0 e^{-i2\pi ft} \tag{1}$$

where p_0 is the amplitude of the turbulent fluctuating pressure, f is the engineering frequency of the turbulent fluctuating pressure, and t is a time parameter. In the above formula, assuming that the average peak value of pulse pressure is approximately $\left(\frac{\rho_w V^2}{2}\right) \times 3\%$, the equation can be rewritten as,

$$p_0 \approx 0.015 \rho_w V^2 \tag{2}$$

where ρ_w is the density of water. Given that the fluctuating pressure frequency is lower for the partition wall and water flow, only the first-order modal control equations governing ξ_1 is considered:

$$\ddot{\xi}_1 + 2\dot{\xi}_1 \zeta \omega_{01} + \omega_{01}^2 \xi_1 = \frac{P_1^*}{M_1} \tag{3}$$

where ζ is the damping ratio and ω_{01} is the circular frequency

of vibration of the structure in the water. $\dot{\xi}$ and $\ddot{\xi}$ are the first- and second-order derivatives of time, respectively. P_1^* and M_1^* are the first vibration modes of the corresponding generalized force and general quality, respectively.

The water partition approximates a cantilever plate. Under the assumption that the wall pressure fluctuations are uniformly distributed, the cantilever plate can approximate the cantilever beam, and the first vibration mode, $Y(\alpha)$, of the beam function is then written as

$$Y(\alpha) = 0.1208 \begin{pmatrix} (\text{sh } r_1 + \sin r_1) & (\text{sh } r_1 \alpha - \cos r_1 \alpha) \\ -(\text{sh } r_1 + \cos r_1) & (\text{sh } r_1 \alpha - \sin r_1 \alpha) \end{pmatrix} \quad (4)$$

where $\alpha = \frac{y}{h}$, r_1 is the coefficient of the first vibration mode of the beam with a value of 1.8751. By substituting r_1 into Eqs. (4)-(6) are obtained as

$$Y(\alpha) = 0.4999(\text{ch } r_1 \alpha - \cos r_1 \alpha) - 0.3670(\text{sh } r_1 \alpha - \sin r_1 \alpha) \quad (5)$$

$$\omega_{01} = 1.8751^2 \sqrt{\frac{EI}{(M_{1C}^* + M_{1W}^*)}} \quad (6)$$

where E is the elastic modulus of the wall materials. I is the wide beam moment of inertia $I = \frac{t^3}{12}$. M_{1C}^* is the corresponding unit beam breadth, and wall quality is the additional quality of the generalized quality unit beam breadth response.

$$M_{1C}^* = \frac{\rho t}{r_1} \int_0^1 Y^2(\alpha) dr_1 \alpha = 0.2505 \rho t h \quad (7)$$

where ρ is the density of the beam. If a solution similar to the ‘‘Westergaard’’ approach is adopted, the additional quality of the generalized quality unit beam breadth response is M_{1W}^* .

$$M_{1W}^* = \frac{7K}{8} \rho_w \int_0^1 \sqrt{1 - \sigma} Y^2(\alpha) d\alpha = 0.092K \rho_w h^2 \quad (8)$$

When water is present on the beam side, $K = 1$ and when the beam has water on both sides, $K = 2$.

$$P_{01}^* = \frac{K p h}{r_1} \int_0^1 \sqrt{1 - \alpha} Y^2(\alpha) d\alpha = 0.3913 K p_0 h \quad (9)$$

When the amplitude of the harmonic vibration is x_0 , the input power and the consumption of damping power are balanced.

Given that the frequency of the input load is ω and the structural natural vibration frequency is equal to ω_{01} , $x_0 = (x_0)_{\max}$ and is also equal to the advance force of the displacement of the phase difference. A new equation can therefore be derived as

$$\begin{cases} W_p = \int_0^T P_{01}^* \sin(\omega_{01} t) \dot{x} dt = \int_0^{\frac{2\pi}{\omega_{01}}} P_{01}^* \sin(\omega_{01} t) (x_0)_{\max} \cos(\omega_{01} t - \varphi) dt \\ W_r = \int_0^T P_r \dot{x} dt = \int_0^{\frac{2\pi}{\omega_{01}}} 2\zeta \omega_{01} (M_{1W}^* + M_{1C}^*) (x_0)_{\max}^2 \omega_{01}^2 \cos^2(\omega_{01} t - \varphi) dt \\ W_p = W_r \\ \varphi = 90^\circ \end{cases} \quad (10)$$

By solving Eq. (10), we have

$$(x_0)_{\max} = \frac{P_{01}^*}{2\zeta \omega_{01}^2 (M_{1C}^* + M_{1W}^*)} \quad (11)$$

If wall height $h = 20$ m, wall thickness is 2 m, density $\rho = 2400$ kg/m³, $E = 2 \times 10^{10}$ N/m², and if on both sides of the wall there is water, the flow rate $V = 20$ m/s, $\zeta = 0.05$, the calculated value of unit width is

$$P_{01}^* = 0.3913 \times 2 \times 0.015 \times 1000 \times 400 \times 20 = 93912 \text{ N}$$

$$M_{1C}^* = 0.2505 \times 2400 \times 2 \times 20 = 24048 \text{ kg}$$

$$M_{1W}^* = 0.092 \times 2 \times 1000 \times 400 = 73600 \text{ kg}$$

$$\omega_{01} = 1.8751^2 \sqrt{\frac{2 \times 10^{10} \times 2^3}{12 \times (24048 + 73600) \times 20^3}} = 14.53 \text{ Rad/s}$$

$$(x_0)_{\max} = 0.046 \text{ m}$$

If the wall thickness is increased from 2 m to 3 m, M_{1C}^* increases to 36072 kg, ω_{01} increases to 25.18 rad/s, and x_0 falls to 0.014 m.

III. RIVER ARCH DAM DISCHARGE VIBRATION ESTIMATES

An overflow of an arch dam section and cross section is depicted in Fig. 2. The top arc length is 255.5 m, the dam height is 235 m, the thickness of the dam crown roof is 2 m, and the thickness of the bottom of the arch crown is 22 m. The discharge section length is approximately 80.5 m and is basically located in the central part of the dam body.

A natural vibration frequency of 6.60 Hz and damping ratio of 2.7% was observed from the prototype, measuring a water level of 217 m. The dam section is represented in Fig. 3(a) and the first vibration mode is illustrated in Fig. 3(b).

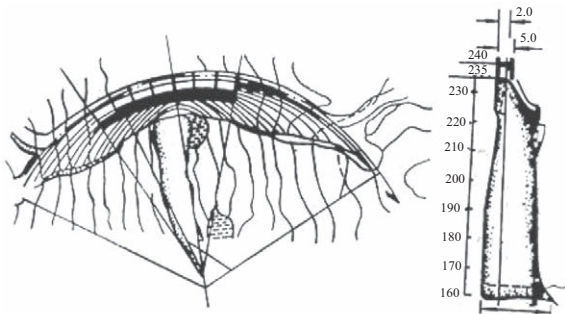
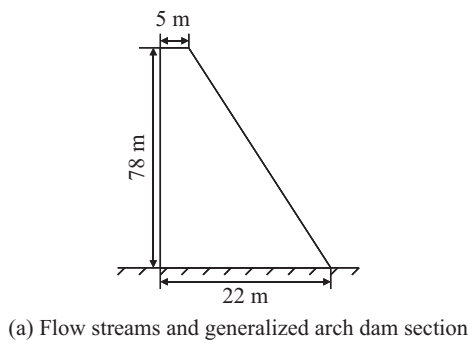


Fig. 2. Overflow of springs on an arch dam section and cross section.



(b) The first vibration mode is generalized

Fig. 3. Arch dam section and first modal generalized diagram.

With the first vibration mode of the corresponding single modal quality, the M_{1d}^* approximation for the width is

$$M_{1d}^* = \rho_d \int_0^h \left(\frac{y}{h}\right)^2 (22 - 17\frac{h}{y}) dy \approx 577.2.$$

where ρ_d is the density of dam material (Concrete) and is 2400 kg/m³.

If the “Westergaard” solution is adopted, with the first vibration mode of the corresponding single dam added mass, M_{1W}^* becomes $M_{1W}^* \approx 0.1524\rho_w \frac{7}{8} h^2 = 811.3 T$. where ρ_w is the density of water.

If the overflow surface in the vertical projection is set to 10 m, the overflow surface average velocity to $v = 15$ m/s, and the pulsation pressure amplitude p to $0.03 \rho_w v^2/2$, the fluctuating

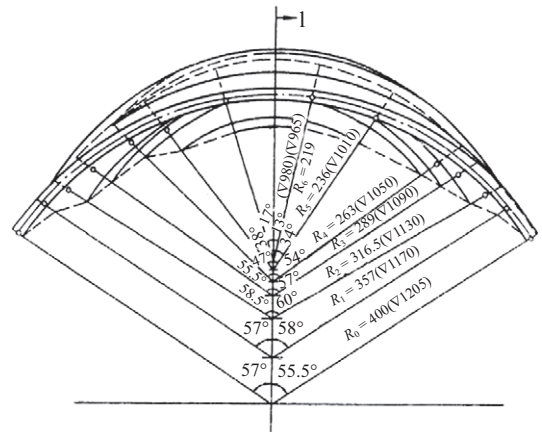


Fig. 4. Layout of the Ertan hyperbolic arch dam project diagrams.

pressure in unit width of the dam is therefore

$$p_1 = 0.03 \times 1000 \times \frac{15^2}{2} \times 10 \times 1 = 33750 \text{ N}.$$

The mode corresponding to the first mode of vibration is

$$p_1^* = p_1 \times 1 = 33750 \text{ N}.$$

Thus, the maximum displacement amplitude vibration $(x_0)_{\max}$ caused by flow-induced vibration is

$$\begin{aligned} (x_0)_{\max} &= \frac{p_1^*}{2\zeta\omega_{01}^2 (M_{1d}^* + M_{1W}^*)} \\ &= \frac{33750}{2 \times 0.027 \times (2\pi \times 6.60)^2 \times (577.2 + 811.3) \times 1000} \\ &= 0.26 \text{ MM} \end{aligned}$$

IV. ESTIMATING THE FLOW-INDUCED VIBRATION OF THE ERTAN HYPERBOLIC ARCH DAM

The maximum dam height of the Ertan hyperbolic arch dam is 240 m. The maximum flood discharge of the dam is 16300 m³/s. The layout of the dam is shown in Fig. 4.

The trajectory energy dissipation is based on the dam body with surface and middle holes, and also the water cushion pool under the dam. The subsidiary dam is of free discharge. The first natural frequency is 1.105 Hz and the damping ratio ζ is 7% when the reservoir is full. Using the same method as the third quarter, an equation is derived as

$$M_{1d}^* = \rho_d \int_0^h \left(\frac{y}{h}\right)^2 (55 - 40\frac{h}{y}) dy = 2.4h \left(\frac{55}{3} - \frac{40}{4}\right) \approx 4800 \text{ T}$$

In this paper, approximation for the following expressions are taken as

$$\frac{M_{1d}^* + M_{1w}^*}{M_{1d}^*} = \left(\frac{1.299}{1.105}\right)^2 = 1.382 \text{ T and}$$

$$M_{1d}^* + M_{1w}^* = 6633.6 \text{ T .}$$

When the overflow surface in the vertical projection is 16 m, the average velocity of the overflow surface becomes $v = 16 \text{ m/s}$. The pressure pulsation amplitude p is 3% $\rho_w v^2/2$. The fluctuating pressure effect on a single side of the dam is given as

$$p_1 = 0.03 \times 1000 \times \frac{16^2}{2} \times 16 \times 1 = 61440 \text{ N ,}$$

$$p_1^* = p_1 \times 0.9 = 55296 \text{ N .}$$

Therefore, the maximum displacement amplitude subjected to the flow-induced vibration caused by the discharge of the surface hole, $(x_0)_{\max}$ is

$$\begin{aligned} (\xi_0)_{\mu\alpha\xi} &= \frac{\pi_1^*}{2\zeta\omega_{01}^2 (M_{1d}^* + M_{1\Omega}^*)} \\ &= \frac{55296}{2 \times 0.07 \times (2\pi \times 1.105)^2 \times 6633600} \\ &= 1.24 \text{ mm} \end{aligned}$$

The Ertan double arch dam underwent an hydro-elastic model test and water discharge prototype observation. When the discharge volume was 6024.7 m³/s, the maximum mean square root value of the radial displacement of the arch dam crest was 19.39 μm . From the hydro-elastic model test results, conversion to the radial displacement of the prototype was 99.6 μm . The calculated vibration value of the Ertan hyperbolic arch dam was slightly greater than those from the results of the hydro-elastic model that was based on experimental and prototype observations. It is a safe and straightforward calculation method despite the recorded observations. For such a problem, the error caused by the result of the calculation is acceptable under general en-

gineering calculation methods.

V. CONCLUSION

Forecasting flow-inducing vibration using model testing is time-consuming. This paper proposes a simple quantitative estimation of flow-induced vibration. The technique produces the flow-induced vibration of water flowing around a dividing wall and that of the Ertan hyperbolic arch dam, and the flood discharge vibration of the Liuxihe arch dam in order to describe the calculation process employed. By comparing the difference between the calculated value of the Ertan hyperbolic arch dam and the prototype observation results, the method proposed in this study offers acceptable results. Moreover, if the vibration estimate of this research model is relatively weaker, it may not be necessary to carry out a model experiment. As a result, this study shows that there are some uncertainties in the value of fluctuating pressure in the usual range, 3%-10% of velocity head. In addition, if the overflow surface has a wide range of spin, further studies would need to be conducted.

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