

# PASSIVE FUZZY CONTROL FOR LIFT FEEDBACK FIN STABILIZER SYSTEMS OF A SHIP VIA MULTIPLICATIVE NOISE BASED ON FUZZY MODEL

Chih-Ming Chang, Wen-Jer Chang, Cheung-Chieh Ku, and Fung-Lin Hsu

Key words: lift feedback fin stabilizer system of a ship, fuzzy control, Takagi-Sugeno fuzzy models and passivity constraint.

## ABSTRACT

This paper proposes a passive fuzzy controller design methodology for stabilization of the nonlinear lift feedback fin stabilizer system of a ship. The proposed design approach is developed based on multiplicative noised Takagi-Sugeno fuzzy model and parallel distributed compensation control technique. Applying the Itô's formula and the sense of mean square, the sufficient conditions are developed to analyze the stability and to design the controller for stochastic nonlinear systems. The sufficient conditions derived in this paper belong to the linear matrix inequality forms which can be solved efficiently by convex optimal programming algorithm. Besides, the passivity theory is applied to discuss the effect of external disturbance on the system. Finally, the proposed systematic design method is applied to the fuzzy controller design of lift-feedback-fin stabilizer systems of a ship. The simulation results of lift-feedback-fin stabilizer control system of a ship show that the proposed passive fuzzy controller design method is effective.

## I. INTRODUCTION

When a ship is sailing on the sea, its violent rolling caused by sea waves may deeply affect the comfort of passengers. Thus, roll motion is one of the most important ship motions. Large amplitude rolling motion would make the crew feel uncomfortable and may cause damage to the cargoes and vessels. In order to reduce the rolling motion of the vessel, the effectiveness of fin stabilizer was investigated (Jin et al., 1994; Tzeng and Wu, 2000; Yu and Liu, 2002; Xiu and Ren, 2003; Jin et al.,

2006; Liang et al., 2008; Perez and Goodwin, 2008; Wang et al., 2008). The fin stabilizer is an effective device widely used in reducing rolling of a ship. It can generate held-up moment initiative to resist sea wave disturbances to reduce rolling of a ship. The PID-based control approach for the fin stabilizer systems was investigated in (Jin et al., 1994). Applying the internal model control method, a ship stabilizing fin controller has been developed in (Tzeng and Wu, 2000). In Yu and Liu (2002), the active disturbance rejection control method was used to find a stabilizing fin controller. Besides, sliding mode control (Jin et al., 2006), model predictive control (Perez and Goodwin, 2008), fuzzy control approach (Xiu and Ren, 2003; Liang et al., 2008; Wang et al., 2008), and Lyapunov's direct method (Karakas et al., 2012), have been employed to deal with the stabilization problem for the nonlinear fin stabilizer systems of a ship, respectively. Extending the PID control technique, some PID-based mixed control methods have been studied in (Liang et al., 2008; Ghassemi and Dadmarzi, 2010) for the fin stabilizer systems. Without loss of generality, lift feedback fin stabilizer is the most effective ship roll reducing equipment. Some control approaches for the lift feedback fin stabilizer system have been investigated in the literature (Xiu and Ren, 2003; Yao et al., 2003; Zhang et al., 2003; Liang et al., 2008; Wang et al., 2008).

Many researchers use the concept of probability to describe the stochastic behavior of systems. The Itô stochastic differential equation (Eli et al., 2005) is also employed to characterize the structure of stochastic systems with multiplicative noises. With Itô stochastic differential equation, the structure of stochastic systems is more representative and understandable. The lift feedback fin stabilizer systems of a ship considered in (Jin et al., 1994; Tzeng and Wu, 2000; Yu and Liu, 2002; Xiu and Ren, 2003; Yao et al., 2003; Zhang et al., 2003; Jin et al., 2006; Liang et al., 2008; Perez and Goodwin, 2008; Wang et al., 2008; Ghassemi and Dadmarzi, 2010; Karakas et al., 2012) did not consider the multiplicative noises. Concerning stochastic process regarding multiplicative noise terms, the processes of differentiation and integration are similar to the analysis of deterministic functions, but they require some extra

care in the evaluation of limits. The detail explanations for evaluation of limits can be referred to (Eli et al., 2005). During past decades, the stability of stochastic systems can be analyzed by the Itô's formula (Eli et al., 2005). Unfortunately, most of the fruitful results are accomplished for the linear stochastic systems. However, many physical systems are nonlinear which usually presented by complex dynamic equations.

The Takagi-Sugeno (T-S) fuzzy model (Takagi and Sugeno, 1985; Tanaka and Wang, 2001) has been attracting increasing attention in the study and application of the stabilization of nonlinear systems. With applying the T-S fuzzy model, the nonlinear systems can be approximated by simple linear subsystems and determined membership functions. According to T-S fuzzy model, the Itô's formula can also be employed to analyze the stability of nonlinear stochastic systems. Besides, the Parallel Distributed Compensation (PDC) concept (Tanaka and Wang, 2001) was employed to design the fuzzy controllers for many nonlinear systems which are modeled by T-S fuzzy models (Teixeira and Zak, 1999; Kim and Kim, 2002; Wang et al., 2004; Chang et al., 2012; Chang and Hsu, 2016; Chiu, 2014; Zhang et al., 2007). However, the nonlinear stochastic systems considered in (Teixeira and Zak, 1999; Kim and Kim, 2002; Wang et al., 2004; Zhang et al., 2007; Chang et al., 2012; Chang and Hsu, 2016) were investigated with more complex limitations without external disturbance effects on systems. Considering the external disturbances, a PDC-based fuzzy control technology is developed in this paper for the nonlinear stochastic systems which are constructed by the stochastic T-S fuzzy models.

In general, the disturbance effect on systems is usually caused by some external factors which may make the system unstable. For attenuating the effect of disturbances, the  $H_\infty$  control scheme (Jeung et al., 1998; Chang and Chang, 2006) was applied to control the linear and nonlinear systems. Besides, the passivity theory (Lozano et al., 2000) was also employed to propose a general form for achieving disturbance attenuation performance. Via different definitions of supply function of passivity, the systems can be stabilized for achieving different types of attenuation performance with requirement energy. In general, the strict input passive type (Li et al., 2005; Chang et al., 2009; Chang et al., 2011; Chang et al., 2013; Chang et al., 2015) is usually used to constrain the disturbance effect and to study the stability of fuzzy systems. Therefore, the concept of strictly input passive property (Li et al., 2005; Chang et al., 2009; Chang et al., 2011; Chang et al., 2013; Chang et al., 2015) is employed to analyze the stability of nonlinear stochastic systems and to achieve attenuation performance in this paper.

Considering the nonlinear lift feedback fin stabilizer system of a ship with multiplicative noises, the fuzzy modeling approach and Itô stochastic differential equation are used in this paper to establish the stochastic T-S fuzzy model. The purpose of this paper is to design a passive fuzzy controller for the nonlinear lift feedback fin stabilizer system of a ship that is constructed by the T-S fuzzy model with multiplicative noises. By using Itô's formula, PDC concept and strictly input passive

theory, the stability conditions of proposed control problem can be derived in term of Linear Matrix Inequality (LMI) (Boyd et al., 1994). The main contribution of this paper is to develop a PDC-based fuzzy control approach, which can be solved by the LMI technique, such that the nonlinear lift feedback fin stabilizer system of a ship can achieve asymptotical stability in mean square and strictly input passivity constraint. In order to demonstrate the effectiveness and application of the proposed fuzzy control method, a control simulation for the nonlinear lift feedback fin stabilizer control system of a ship is provided.

## II. MODEL DESCRIPTIONS AND PROBLEM STATEMENTS

Based on the Teixeira-Zak's formula (Teixeira and Zak, 1999), a nonlinear system can be divided into several local linear subsystems via the membership functions and corresponding operating points. Combining these linear subsystems, a T-S fuzzy model can be constructed by a set of fuzzy IF-THEN rules. The  $i$ -th fuzzy rule of the T-S fuzzy model with multiplicative noises can be described as the following form:

**Rule  $i$ :** IF  $x_1(t)$  is  $M_{i1}$  ... and  $x_n(t)$  is  $M_{in}$  THEN

$$dx(t) = [A_i x(t) + B_{ui} u(t) + B_{wi} w(t)] dt + [\tilde{A}_i x(t)] dq(t) \quad (1a)$$

$$y(t) = C_i x(t) + D_i w(t) \quad (1b)$$

or

$$dx(t) = \sum_{i=1}^r g_i(x(t)) \{ [A_i x(t) + B_{ui} u(t) + B_{wi} w(t)] dt + [\tilde{A}_i x(t)] dq(t) \} \quad (2a)$$

$$y(t) = \sum_{i=1}^r g_i(x(t)) \{ C_i x(t) + D_i w(t) \} \quad (2b)$$

where  $g_i(x(t)) = \frac{\prod_{e=1}^n M_{ie}(x_e(t))}{\sum_{i=1}^r \prod_{e=1}^n M_{ie}(x_e(t))}$  and  $M_{ie}(x_e(t))$  is the

grade of membership function of the  $x_e(t)$  in  $M_{ie}$ ,  $M_{ie}$  is the fuzzy set;  $n$  is the premise variable number;  $A_i$ ,  $B_{ui}$ ,  $B_{wi}$ ,  $\tilde{A}_i$ ,  $C_i$  and  $D_i$  are constant matrices with the compatible dimensions,  $x(t) \in \mathfrak{R}^{n_x}$  is the state vector,  $u(t) \in \mathfrak{R}^{n_u}$  is the input vector,  $y(t) \in \mathfrak{R}^{n_y}$  is the output vector,  $r$  is the number of fuzzy rules,  $w(t) \in \mathfrak{R}^{n_w}$  is the external disturbance input vector which is assumed as a bounded zero mean signal, the

$q(t)$  is a scalar Brownian motion which is defined on complete probability space (Wang et al., 2004). Let us define  $E[Q(a)]$  as the expected value of  $Q(a)$ . By referring the reference (Eli et al., 2005), the properties of  $E\{dq(t)\} = 0$ ,  $E\{x(t)dq(t)\} = E\{x(t)\}E\{dq(t)\} = 0$  and  $E\{w(t)dq(t)\} = E\{w(t)\}E\{dq(t)\} = 0$  are assumed due to the independent increment property of Brownian motion.

Applying the concept of PDC, the fuzzy controller is designed to share the same IF part of the T-S fuzzy model (1). The fuzzy controller can be represented as follows:

**Rule i:** IF  $x_1(t)$  is  $M_{i1}$  and ... and  $x_n(t)$  is  $M_{in}$  THEN

$$u(t) = -F_i x(t) \quad (3)$$

or

$$u(t) = \sum_{i=1}^r g_i(x(t))(-F_i x(t)) \quad (4)$$

Substituting (4) into (2a), the closed-loop T-S fuzzy model with multiplicative noise can be obtained such as

$$dx(t) = \sum_{i=1}^r \sum_{j=1}^r g_i(x(t))g_j(x(t))\{[G_{ij}x(t) + B_{w_{ij}}w(t)]dt + [\tilde{A}_i x(t)]dq(t)\} \quad (5)$$

where

$$G_{ij} = \frac{A_i - B_{m_i}F_j + A_j - B_{m_j}F_i}{2}, B_{w_{ij}} = \frac{B_{w_i} + B_{w_j}}{2}$$

For achieving the attenuating performance, the passivity theory provides a useful and effective tool to design the controller to achieve the energy constraints for the closed-loop systems. In the passivity theory, the supply rate is an important role in determining the kind of energy change. In order to constrain the disturbance energy, the strict input passivity is introduced in the following definition.

**Definition 1 (Chang et al., 2009)**

The system (5) with external disturbance  $w(t)$  and output  $y(t)$  is called strictly input passive if there exists a positive scalar  $\beta$  and symmetric positive definite matrix  $U = U^T > 0$  such that

$$E\left\{2 \int_0^{t_p} y^T(t)Uw(t) dt\right\} > E\left\{\beta \int_0^{t_p} w^T(t)w(t) dt\right\} \quad (6)$$

for all  $t_p \geq 0$  and  $w(t) \neq 0$ . The  $t_p > 0$  is the terminal time and

the  $U$  is a constant matrix with compatible dimensions.

Matrix  $U$  is chosen as an identity matrix to describe dissipative energy. Moreover, the scalar  $\beta$  is chosen to determine the dissipative rate for disturbance input. Considering the T-S fuzzy model (1) with multiplicative noises, the purpose of this paper is to find a PDC-based fuzzy controller (3) such that the closed-loop system (5) is stable and the passivity constraint defined in Definition 1 is achieved. In the following section, some sufficient conditions are derived for finding the above passive PDC-based fuzzy controllers.

**III. PDC-BASED FUZZY CONTROLLER DESIGN WITH PASSIVITY CONSTRAINT**

The PDC-based fuzzy controller design for T-S fuzzy models with multiplicative noise is developed in this section. The sufficient conditions for guaranteeing the stability and passivity constraint of closed-loop T-S fuzzy models are derived based on the Lyapunov theory and passivity theory. According to the closed-loop T-S fuzzy model (5), the stability conditions are derived in the following theorem.

**Theorem 1**

If there exist symmetric positive definite matrices  $P = P^T > 0$  and  $U = U^T > 0$ , feedback gains  $F_i$ , and dissipation rate  $\beta$  satisfying the following stability conditions, then the closed-loop T-S fuzzy system (5) is strictly input passive and asymptotically stable in mean square.

$$\begin{bmatrix} G_{ij}^T P + P G_{ij} + \tilde{A}_i^T P \tilde{A}_i & P B_{w_{ij}} - C_i^T U \\ * & \beta I - D_i^T U - U^T D_i \end{bmatrix} < 0 \quad (7)$$

where \* denotes the transposed elements or matrices for symmetric position and  $I$  is the identity matrix with compatible dimension.

**Proof:**

To analyze the stability of the closed-loop T-S fuzzy system (5), a Lyapunov function is chosen as  $V(x(t)) = x^T(t)Px(t)$ .

The derivative of  $V(x(t))$  along the trajectory of (5) can be obtained by Itô's formula (Tanaka and Wang, 2001; Eli et al., 2005) such as

$$dV(x(t)) = LV(x(t))dt + \theta(x(t))dq(t) \quad (8)$$

where

$$LV(x(t)) = \sum_{i=1}^r \sum_{j=1}^r g_i(x(t))g_j(x(t))\{x^T(t)(G_{ij}^T P + P G_{ij} + \tilde{A}_i^T P \tilde{A}_i)x(t) + 2x^T(t)(P B_{w_{ij}})w(t)\} \quad (9a)$$

$$\theta(x(t)) = 2x^T(t) \mathbf{P} \sum_{i=1}^r \sum_{j=1}^r g_i(x(t)) g_j(x(t)) (\tilde{\mathbf{A}}_i x(t)) \quad (9b)$$

Arranging the (9a), one has

$$LV(x(t)) = \sum_{i=1}^r \sum_{j=1}^r g_i(x(t)) g_j(x(t)) \begin{bmatrix} x(t) \\ w(t) \end{bmatrix}^T \times \begin{bmatrix} \mathbf{G}_{ij}^T \mathbf{P} + \mathbf{P} \mathbf{G}_{ij} + \tilde{\mathbf{A}}_i^T \mathbf{P} \tilde{\mathbf{A}}_i & \mathbf{P} \mathbf{B}_{wij} \\ * & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} \quad (10)$$

Let us take the expectation of (8), then one has the following equation with the independent increment property of Brownian motion (Eli et al., 2005), i.e.,  $E\{\theta(x(t))dq(t)\} = E\{\theta(x(t))\}E\{dq(t)\} = 0$  with  $E\{dq(t)\} = 0$ .

$$E\{dV(x(t))\} = E\{LV(x(t))dt\} \quad (11)$$

Integrating both sides of (11) from 0 to  $t_p$ , one has the following equation with zero initial condition.

$$E\{V(x(t_p))\} = E\left\{\int_0^{t_p} LV(x(t))dt\right\} \quad (12)$$

For nonzero external disturbance, i.e.,  $w(t) \neq 0$ , one can define a performance function such as

$$\begin{aligned} \eta(x, w, t) &= E\left\{\int_0^{t_p} \beta w^T(t)w(t) - 2y^T(t)Uw(t) dt\right\} \\ &= E\left\{\int_0^{t_p} [\beta w^T(t)w(t) - 2y^T(t)Uw(t) + LV(x(t))] dt - V(x(t_p))\right\} \\ &\leq E\left\{\int_0^{t_p} \lambda(x, w, t) dt\right\} \end{aligned} \quad (13)$$

where

$$\lambda(x, w, t) = \beta w^T(t)w(t) - 2y^T(t)Uw(t) + LV(x(t)) \quad (14)$$

Substituting (2b) and (10) into (14), one can obtain

$$\lambda(x, w, t) = \sum_{i=1}^r \sum_{j=1}^r g_i(x(t)) g_j(x(t)) \begin{bmatrix} x(t) \\ w(t) \end{bmatrix}^T \mathbf{T} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} \quad (15)$$

where

$$\mathbf{T} = \begin{bmatrix} \mathbf{G}_{ij}^T \mathbf{P} + \mathbf{P} \mathbf{G}_{ij} + \tilde{\mathbf{A}}_i^T \mathbf{P} \tilde{\mathbf{A}}_i & \mathbf{P} \mathbf{B}_{wij} - \mathbf{C}_i^T \mathbf{U} \\ * & \beta \mathbf{I} - \mathbf{D}_i^T \mathbf{U} - \mathbf{U}^T \mathbf{D}_i \end{bmatrix}$$

If the condition (7) is satisfied, then one can obtain  $T < 0$  that implies  $\lambda(x, w, t) < 0$ . From (13), the inequality  $\lambda(x, w, t) < 0$  implies

$$\eta(x, w, t) < 0 \quad (16)$$

or

$$E\left\{2 \int_0^{t_p} y^T(t)Uw(t)dt\right\} > E\left\{\beta \int_0^{t_p} w^T(t)w(t)dt\right\} \quad (17)$$

Since (17) is equivalent to (6), the system is strictly input passive.

Next, it is necessary to show that the system is asymptotically stable in mean square. According to (15), if the condition (7) is satisfied, i.e.,  $T < 0$ , then one has  $\lambda(x, w, t) < 0$ . By assuming  $w(t) = 0$ , one can find  $LV(x(t)) < 0$  from (14) due to  $\lambda(x, w, t) < 0$ . Since  $w(t) = 0$  and  $LV(x(t)) < 0$ , one can obtain the following equation from (11).

$$E\{dV(x(t))\} = E\{LV(x(t)) dt\} < 0 \quad (18)$$

Based on the Lemma 6.1 of (Eli et al., 2005), one can find that the system is asymptotically stable in mean square driven by control law (4). The proof of this theorem is completed.

Applying the passivity theory and Itô's formula, the condition (7) is developed to analyze the stability of the closed-loop T-S fuzzy model (5). However, the condition (7) cannot be calculated by LMI technique. Thus, the concept of the Schur Complement (Boyd et al., 1994) is employed to convert the bilinear matrix inequality condition (7) into the LMI problem in the following theorem.

**Theorem 2**

If there exist positive definite matrices  $\mathbf{P} = \mathbf{P}^T > 0$  and  $\mathbf{U} > 0$ , feedback gains  $\mathbf{F}_i$  and dissipative rate  $\beta$  satisfying the following conditions, then the closed-loop T-S fuzzy system (5) is strictly input passive and asymptotically stable in mean square.

$$\begin{bmatrix} \frac{1}{2}(\mathbf{X} \mathbf{A}_i^T - \mathbf{Y}_j^T \mathbf{B}_{ii}^T + \mathbf{A}_i \mathbf{X} - \mathbf{B}_{ii} \mathbf{Y}_j) & \mathbf{B}_{wij} - \mathbf{X} \mathbf{C}_i^T \mathbf{U} & \mathbf{X} \tilde{\mathbf{A}}_i^T \\ * & \beta \mathbf{I} - \mathbf{D}_i^T \mathbf{U} - \mathbf{U}^T \mathbf{D}_i & 0 \\ * & * & -\mathbf{X} \end{bmatrix} < 0 \quad (19)$$

where  $\mathbf{X} = \mathbf{P}^{-1}$  and  $\mathbf{Y}_i = \mathbf{F}_i \mathbf{X}$ .

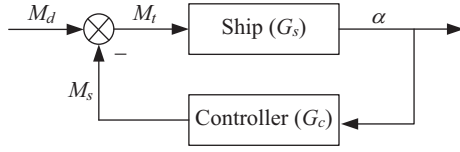


Fig. 1. The block diagram of lift feedback fin stabilizer control system of a ship.

**Proof:**

Multiplying both sides of (7) by  $diag\{P^{-1}, I\}$ , the LMI condition (19) can be easily derived by applying the Schur complement (Boyd et al., 1994) and setting new variables, i.e.,  $X = P^{-1}$  and  $Y_i = F_i X$ . Due to the derivations of condition (19) is well known, the detailed proof is omitted here.

Based on the condition of Theorem 2, the fuzzy control gains can be obtained via LMI technique by using MATLAB LMI-Toolbox. Hence, the fuzzy controller for T-S fuzzy models with multiplicative noise (5) can be designed by PDC concept with solving the LMI conditions (19).

**IV. PASSIVE FUZZY CONTROL FOR LIFT FEEDBACK FIN STABILIZER SYSTEMS OF A SHIP**

A lift feedback fin stabilizer control system of a ship consists of a controller, sensor, servo system, fin and etc. (Xiu and Ren, 2003). The lift feedback fin encapsulates the nonlinear and uncertain relationship between lift and fin angle, and all the other parts are linear blocks. Thus, all parts except ship can be simplified and united to a controller block when a system is designed. The block diagram of lift feedback fin stabilizer control system of a ship can be shown in Fig. 1.

Rolling of a ship is mainly caused by sea waves. Sea waves have statistical rule although they are irregular and stochastic. In this paper, the main energy of sea waves centralizes in the low-frequency range of 0.3-1.25 rad/s by sea wave spectrum theory. Considering the nonlinearities of rolling resilient moment and rolling damp moment, we can describe the nonlinear model of rolling of a ship as follows:

$$(I_x + \Delta I_x)\ddot{\alpha} + B_1\dot{\alpha} + B_2|\dot{\alpha}|\dot{\alpha} + C_1\alpha + C_3\alpha^3 + C_5\alpha^5 = M_d - M_s \tag{20}$$

where  $\alpha$  denotes the rolling angle of a ship,  $I_x$  and  $\Delta I_x$  denote the mass inertia moment and affixing mas inertia moment relative to the vertical axes of the ship,  $M_d$  denotes the disturbance moment of sea waves,  $M_s$  denotes the stable moment of lift feedback fins. Besides,  $C_1, C_3, C_5, B_1, B_2$  are constants and  $C_1 = Dh$ , where  $D$  denotes the tonnage of a ship and  $h$  denotes the height of the steady center of rolling of a ship.

The parameters of a certain ship are  $D = 1457.26$  t,  $h = 1.15$

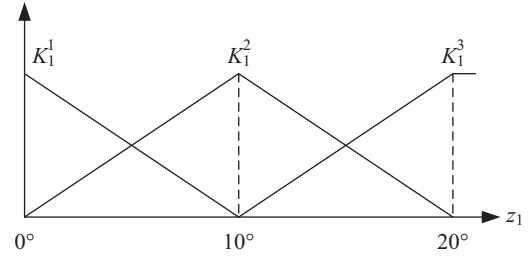


Fig. 2. The membership functions of  $K_1^i$ .

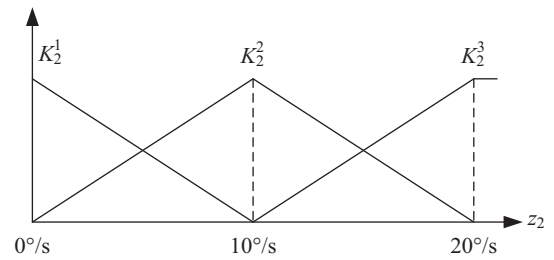


Fig. 3. The membership functions of  $K_2^j$ .

$m, I_x + \Delta I_x = 3.4383 \times 10^6, C_3 = 2.097 \times 10^6, C_5 = 4.814 \times 10^6, B_1 = 0.636 \times 10^6, B_2 = 0.79 \times 10^6$ . Let  $M_t = M_d \times M_s$ , the nonlinear model of this rolling of a ship can be described as follows:

$$\ddot{\alpha} + 0.185\dot{\alpha} + 0.23|\dot{\alpha}|\dot{\alpha} + 0.4874\alpha + 0.61\alpha^3 + 1.4\alpha^5 = 2.9084 \times 10^{-7} M_t \tag{21}$$

The nonlinear model of rolling of a ship described as (21) can be approximated by a T-S fuzzy model as follows:

$$dx(t) = \sum_{i=1}^9 g_i(x(t)) \left\{ \left[ A_i x(t) + B_{in} u(t) + B_{wi} w(t) \right] dt + \left[ \tilde{A}_i x(t) \right] dq(t) \right\} \tag{22}$$

Besides, the output variables are described as follows:

$$y(t) = Cx(t) + Dw(t) \tag{23}$$

where the system state vector is  $x = [x_1, x_2]^T$ ,  $x_1 = \alpha$ ,  $x_2 = \dot{\alpha}$ ,  $u = M_t$ ,  $z_1 = |x_1|$ ,  $z_2 = |x_2|$  are input variables. The type of membership function may not enormously influence the performance of fuzzy controllers. In this paper, a triangular type membership function is chosen to merge the obtained feedback gains. Thus, the corresponding fuzzy sets for  $\{K_1^i, i = 1, 2, 3\}$

and  $\{K_2^j, j=1, 2, 3\}$  are triangular membership functions that are shown in Fig. 2 and Fig. 3, respectively.

The parameter matrices of local linear subsystems are described as follows:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -0.4874 & -0.185 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ -0.5073 & -0.185 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 1 \\ -0.5272 & -0.185 \end{bmatrix}, A_4 = \begin{bmatrix} 0 & 1 \\ -0.4874 & -0.225 \end{bmatrix},$$

$$A_5 = \begin{bmatrix} 0 & 1 \\ -0.5073 & -0.225 \end{bmatrix}, A_6 = \begin{bmatrix} 0 & 1 \\ -0.5272 & -0.225 \end{bmatrix},$$

$$A_7 = \begin{bmatrix} 0 & 1 \\ -0.4874 & -0.265 \end{bmatrix}, A_8 = \begin{bmatrix} 0 & 1 \\ -0.5073 & -0.265 \end{bmatrix},$$

$$A_9 = \begin{bmatrix} 0 & 1 \\ -0.5272 & -0.265 \end{bmatrix}, B_u = \begin{bmatrix} 0 \\ 2.9084 \times 10^{-7} \end{bmatrix},$$

$$B_w = \begin{bmatrix} 0 \\ 0.05 \end{bmatrix}, \tilde{A} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{\pi}{45} \end{bmatrix}, C = [1 \ 0] \text{ and } D = 1.$$

For starting analyzing and designing, one can choose the supply rate  $\beta = 1$  and  $U = 1$ . Solving the sufficient condition of (19) via MATLAB LMI-Toolbox, the matrix  $P = P^T > 0$  can be obtained as follows:

$$P = \begin{bmatrix} 2.9831 & 0.519 \\ 0.519 & 1.0747 \end{bmatrix} \quad (24)$$

Let  $F_i$  denoted the state feedback gain of the local  $i$ th model. Thus, the corresponding PDC-based fuzzy controller can be obtained as follows:

$$u(t) = \sum_{i=1}^9 g_i(x(t))(-F_i x(t)) \quad (25)$$

where the controller gains are solved as follows:

$$F_1 = [147.0585 \ 69.0424],$$

$$F_2 = [146.1067 \ 69.0424],$$

$$F_3 = [145.1549 \ 69.0424],$$

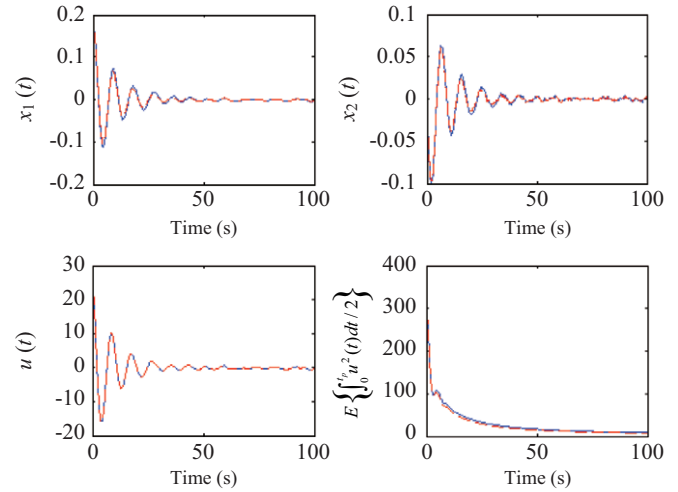


Fig. 4. Responses of nonlinear model of rolling of a ship (Real system: blue solid, T-S fuzzy mode: red dashed).

$$F_4 = [147.0585 \ 67.1293],$$

$$F_5 = [146.1067 \ 67.1293],$$

$$F_6 = [145.1549 \ 67.1293],$$

$$F_7 = [147.0585 \ 65.2162],$$

$$F_8 = [146.1067 \ 65.2162],$$

$$F_9 = [145.1549 \ 65.2162].$$

To assess the effectiveness of the PDC controller, we apply the controller to control the original lift feedback fin stabilizer system of a ship. Simulations indicate that the control law can balance the lift feedback fin stabilizer system of a ship with the multiplicative noises. Moreover, the simulation results are presented in Fig. 4, in which the blue The Fig. 4 shows the responses by each degree of the ship and its corresponding time. The results in Fig. 4 are obtained with the initial condition  $x(0) = [\frac{\pi}{18} \ 0]^T$ .

In the simulations, the external disturbance  $w(t)$  is chosen as a zero mean white noise with variance one. From the simulation results, the effect of the external disturbance on the proposed system can be criticized as follows:

$$\frac{E \left\{ 2 \int_0^{t_p} y^T(t) U w(t) dt \right\}}{E \left\{ \int_0^{t_p} w^T(t) w(t) dt \right\}} = 2.0334 \quad (26)$$

The ratio value of (26) is bigger than determined dissipation rate  $\beta = 1$ , one can find that the condition (6) of Definition 1 is satisfied. Therefore, the considered nonlinear lift feedback fin stabilizer fuzzy control system of a ship (22) with multiplicative noises can achieve asymptotical stability in mean square and strict input passivity constraint by the proposed fuzzy controller (25). Therefore, the systematic design and stability analysis of a lift feedback fin stabilizer fuzzy control system of a ship is validated to be effective in this paper.

## V. CONCLUSIONS

In this paper, the passive fuzzy controller design methodology for a nonlinear lift feedback fin stabilizer system of a ship was studied. The considered nonlinear system was constructed by a stochastic T-S fuzzy model with multiplicative noises. Applying passivity theory and Itô's formula, the external disturbance and multiplicative noise characteristics can be analyzed by the energy concept and the stochastic differential equation. Based on the PDC concept, the proposed fuzzy controller design approach was carried out by solving the LMI stability conditions. Therefore, the control design problem can be solved efficiently in practice by MATLAB LMI-Toolbox and convex programming techniques. In the numerical example, a lift feedback fin stabilizer system of a ship was introduced to illustrate the usefulness and effectiveness of the proposed fuzzy control design methodology.

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