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EFFICIENCY MEASURES FOR VRM MODELS DEALING WITH NEGATIVE DATA IN DEA

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ABSTRACT

This paper explores the properties of the efficiency measures of the variant of radial measure (VRM) which is capable of dealing with negative data in DEA. We found that the input-oriented efficiency measure for VRM might be negative and the range of the output-oriented efficiency measure for VRM might be limited to [0.5, 1]. To circumvent these two situations, we propose the new efficiency measures which prevent the input-oriented efficiency from becoming negative and the output-oriented efficiency from being limited to [0.5, 1].

I. INTRODUCTION

In the standard data envelopment analysis (DEA) approach, all inputs and outputs data are assumed to be non-negative. Lovell and Pastor (1995) and Pastor (1996) were the first to point out that using the translation invariance discovered by Ali and Seiford (1990), negative data can be treated under the assumption of variable returns to scale (VRS). The key to treating negative data in DEA lies in the translation invariance property due to the convexity constraint in the VRS DEA models. The translation invariance property allows the user to move the origin of the data so that all decision making units (DMUs) are in the first quadrant. The constant returns to scale (CRS) DEA models (e.g., the CCR model of Charnes et al. (1978)) do not have the translation invariance property. Therefore, negative data cannot be directly used under any CRS DEA models.

The standard VRS DEA models can be used after the negative data are translated into positive ones. While the classification of efficient and inefficient DMUs remains unchanged after the data translation, depending on a VRS model's orientation and on whether we have negative input and/or output data, scores for inefficient DMUs may not be translation invariant (e.g., Seiford and Zhu, 2002). In addition to the standard VRS model, many of the newly developed DEA approaches for negative data are developed under the assumption of VRS, due to the translation invariance property. For example, based upon Chambers et al.'s (1996, 1998) directional distance model, Portela et al. (2004) developed a range-adjusted model when some of the data are negative. Sharp et al. (2007) built a slacks-based model of Tone (2001) for situations when both negative inputs and outputs are present.

Scheel (2001), on the other hand, proposed to treat absolute values of negative inputs as outputs and absolute values of negative outputs as inputs, namely a smaller negative input value is converted into a larger output value and a larger negative value is converted into a smaller input value. Scheel's (2001) approach does not require the VRS condition. However, its results are expected to be different, because the actual inputs and outputs are different, namely, the roles of inputs and outputs are switched.

Emrouznejad et al. (2010) discussed advantages and disadvantages of the abovementioned approaches for treating negative data in DEA. They developed a semi-oriented radial measure (SORM) when some DMUs have negative inputs/outputs. The SORM method has the advantage that the negative value can be dealt without changing the origin. The preservation of the origin means that a form of radial frontier projection can be pursuit without the need for data translation. Later, Cheng et al. (2013) developed a variant of the traditional radial model whereby original values are replaced with absolute values as the basis to quantify the proportion of improvements to reach the frontier. The new radial measure, the variant of the radial measure (VRM), can be dealt with the presence of negative data. Recently, Kerstens and Van de Woestyne (2014) commented that input-oriented measure of the VRM might be negative and the VRM model should assume variable returns to scale.

This paper further explores the properties of the efficiency measures for VRM. We find that the efficiencies given by the output-oriented efficiency measure for VRM proposed by Cheng et al. (2013) might be limited to the range [0.5, 1]. Kerstens and Woestyne (2014) noted that the efficiencies given by the VRM input-oriented measure proposed by Cheng et al. (2013) might

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be negative. To resolve these two situations, we propose new efficiency measures for input-oriented and output-oriented VRM models.

II. THE VARIANT OF THE RADIAL MEASURE (VRM) MODEL

Suppose we have a set of n DMUs, $\{DMU_j: j = 1, 2, \ldots, n\}$. Assume that there are m inputs and s outputs for each DMU. The ith input of the jth DMU is denoted as x_{ij} and the rth output of the jth DMU is denoted as y_{rj}.

The input-oriented VRS model when the kth DMU is under evaluation can be expressed as (Banker et al., 1984):

$$\begin{aligned} \min \quad & \theta \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{ik},\ i = 1, \ldots, m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rk},\ r = 1, \ldots, s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \ge 0,\ j = 1, \ldots, n \end{aligned} \tag{1}$$

The output-oriented model can be written as:

$$\begin{aligned} \max \quad & \phi \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{ik},\ i = 1, \ldots, m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \ge \phi y_{rk},\ r = 1, \ldots, s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \ge 0,\ j = 1, \ldots, n \end{aligned} \tag{2}$$

By replacing θ with $1 - \beta$ and ϕ with $1 + \gamma$, model (1) and model (2) can be equivalently transformed into model (3) and model (4), respectively:

$$\begin{aligned} \max \quad & \beta \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + \beta x_{ik} \le x_{ik},\ i = 1, \ldots, m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rk},\ r = 1, \ldots, s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \ge 0,\ j = 1, \ldots, n \end{aligned} \tag{3}$$

and

$$\begin{aligned} \max \quad & \gamma \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{ik},\ i = 1, \ldots, m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - \gamma y_{rk} \ge y_{rk},\ r = 1, \ldots, s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \ge 0,\ j = 1, \ldots, n \end{aligned} \tag{4}$$

To deal with negative data, Cheng et al. (2013) transformed model (3) into model (5) and model (4) into model (6). Model (5) is for the transformed DEA model with input-oriented while the model (6) is for the output-oriented one:

$$\begin{aligned} \max \quad & \beta \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + \beta |x_{ik}| \le x_{ik},\ i = 1, \ldots, m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rk},\ r = 1, \ldots, s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \ge 0,\ j = 1, \ldots, n \end{aligned} \tag{5}$$

$$\begin{aligned} \max \quad & \gamma \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{ik},\ i = 1, \ldots, m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - \gamma |y_{rk}| \ge y_{rk},\ r = 1, \ldots, s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \ge 0,\ j = 1, \ldots, n \end{aligned} \tag{6}$$

Note that β^* and γ^* can measure the inefficiency, i.e., the distance from the evaluated DMU to the efficient frontier.

III. THE EFFICIENCY MEASURE OF VRM

For DMU-k, Cheng et al. (2013) defined the input-oriented efficiency given by VRM model (5) to be $VRM^i = 1 - \beta^*$ and the output-oriented efficiency given by VRM model (6) to be $VRM^o = \dfrac{1}{1+\gamma^*}$ respectively.

However, β^* might be larger than 1 and hence $1 - \beta^*$ might be negative as noted in Kerstens and Woestyne (2014). Kerstens and Woestyne (2014) also noted the following property:

Property 1. $0 \leq 1 - \beta^* \leq 1$ if at least one of the input dimensions is strictly positive.

Property 2. $0 \leq \frac{1}{1+\gamma^*} \leq 1$.

We find additional properties for VRM as follows:

Property 3. $\frac{1}{2} \leq \frac{1}{1+\gamma^*} \leq 1$ if at least one of the output dimensions is non-positive, which means that there exists r such that $y_{rj} \leq 0$ for all $j = 1, \ldots, n$.

Proof

Assume that the rth output of all DMUs is non-positive. That is, $y_{rj} \leq 0$ for $j = 1, \ldots, n$. $\sum_{j=1}^{n} \lambda_j y_{rj} \leq 0$ $0 \geq \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rk}$

$$0 > \sum_{j=1}^{n} \lambda_j y_{rj} - \gamma |y_{rk}| \geq y_{rk}, \ \gamma \leq 1, \ 1+\gamma \leq 2, \ \frac{1}{1+\gamma} \geq \frac{1}{2} \qquad \square.$$

Property 4. $0 \leq \frac{1}{1+\beta^*} \leq 1$

Property 5. $\frac{1}{2} \leq \frac{1}{1+\beta^*} \leq 1$ if at least one of the input dimensions is nonnegative.

Property 6. $0 \leq 1-\gamma^* \leq 1$ if at least one of the output dimensions is non-positive.

To avoid that input-oriented efficiency becomes negative, we define the new efficiency measure, $NVRM^i$, for the input-oriented VRM model as:

$$NVRM^i = \begin{cases} 1-\beta^* & \text{if one of the input dimensions is nonnegative} \\ \frac{1}{1+\beta^*} & \text{otherwise} \end{cases}$$

Obviously, according to property 1, $NVRM^i$ will be nonnegative. The reasoning for the definition of $NVRM^i$ is as follows. The larger β^*, the lesser the efficiency. We have two choices for the definition of the input-oriented efficiency, $NVRM^i$. One is $1 - \beta^*$, and the other is $\frac{1}{1+\beta^*}$. If we define $NVRM^i$ in terms of $\frac{1}{1+\beta^*}$, $NVRM^i$ will be nonnegative. According to property 5, if there exits input dimension which is nonnegative, $NVRM^i$ will be confined to the interval [0.5, 1]. Under such circumstance, we define $NVRM^i$ as $1 - \beta^*$. Therefore, we define $NVRM^i$ as $1 - \beta^*$ when there exists input dimension which is nonnegative, and $NVRM^i$ as $\frac{1}{1+\beta^*}$ otherwise.

Likewise, to avoid the spread of efficiencies limited to the range between 0.5 and 1 when some of the output dimensions are non-positive, the new output-oriented efficiency measure $NVRM^o$ is now given by

$$NVRM^o = \begin{cases} 1-\gamma^* & \text{if one of the output dimensions is nonpositive} \\ \frac{1}{1+\gamma^*} & \text{otherwise} \end{cases}$$

Table 1. National effluent processing system example from Emrouznejad et al. (2010).

DMU	(I_1) Cost	(I_2) Effluent	(O_1) Saleable	(O_2) CO_2	(O_3) Methane
1	1.03	-0.05	0.56	-0.09	-0.44
2	1.75	-0.17	0.74	-0.24	-0.31
3	1.44	-0.56	1.37	-0.35	-0.21
4	10.8	-0.22	5.61	-0.98	-3.79
5	1.30	-0.07	0.49	-1.08	-0.34
6	1.98	-0.10	1.61	-0.44	-0.34
7	0.97	-0.17	0.82	-0.08	-0.43
8	9.82	-2.32	5.61	-1.42	-1.94
9	1.59	0.00	0.52	0.00	-0.37
10	5.96	-0.15	2.14	-0.52	-0.18
11	1.29	-0.11	0.57	0.00	-0.24
12	2.38	-0.25	0.57	-0.67	-0.43
13	10.3	-0.16	9.56	-0.58	0.00

IV. NUMERICAL EXAMPLES

To demonstrate that our new efficiency measure avoids the negative efficiency in input-oriented VRM model and limited efficiency range in the output-oriented VRM one, the examples in Kerstens and Woestyne (2014) are revisited in this study. The researched data set adopted form Emrouznejad et al. (2010) is shown in Table 1. There are two inputs (I_1, I_2) and three outputs (O_1, O_2, O_3) in the data set. Kerstens and Woestyne (2014) generated two example cases from the data set in Table 1. In Case 1, there are two inputs (I_1, I_2), one of which is positive (I_1), and three outputs (O_1, O_2, O_3), two of which are non-positive (O_2, O_3).

The efficiencies in Case 1 for the input-oriented VRM measure VRM^i, and the output-oriented VRM measure VRM^o, are shown in Table 2. As we can see from the VRM^o column in Table 2, all efficiencies are greater than 0.5, which is not accidental. Because the two of the output dimensions are non-positive, according to property 3, we have $\frac{1}{2} \leq \frac{1}{1+\gamma^*} \leq 1$. Hence $\frac{1}{2} \leq VRM^o = \frac{1}{1+\gamma^*} \leq 1$. Applying our new efficiency

Table 2. VRM efficiencies for Case 1 where the inputs are I_1 and I_2, and the outputs are O_1, O_2, and O_3.

DMU	β^*	γ^*	VRM^i	VRM^o
1	0.0583	0.1038	0.9417	0.9060
2	0.3188	0.2995	0.6812	0.7695
3	0.0000	0.0000	1.0000	1.0000
4	0.4420	0.4626	0.5580	0.6837
5	0.1372	0.2977	0.8628	0.7706
6	0.1416	0.1610	0.8584	0.8613
7	0.0000	0.0000	1.0000	1.0000
8	0.0000	0.0000	1.0000	1.0000
9	0.1887	0.0961	0.8113	0.9123
10	0.5460	0.3693	0.4540	0.7303
11	0.0000	0.0000	1.0000	1.0000
12	0.4899	0.5504	0.5101	0.6450
13	0.0000	0.0000	1.0000	1.0000

Table 3. VRM efficiencies for Case 2 where the inputs is I_2, and the outputs are O_1, O_2, and O_3.

DMU	β^*	γ^*	VRM^i	VRM^o
1	4.0014	0.4970	-3.0014	0.6680
2	1.5746	0.3822	-0.5746	0.7235
3	0.0000	0.0000	1.0000	1.0000
4	5.5595	0.4626	-4.5595	0.6837
5	8.8893	0.6949	-7.8893	0.5900
6	5.9925	0.5404	-4.9925	0.6492
7	0.3795	0.2639	0.6205	0.7912
8	0.0000	0.0000	1.0000	1.0000
9	1.0000	0.0961	0.0000	0.9123
10	2.3524	0.3841	-1.3524	0.7225
11	0.0000	0.0000	1.0000	1.0000
12	2.1353	0.6150	-1.1353	0.6192
13	0.0000	0.0000	1.0000	1.0000

measure to Case 1, as shown in Table 4, we have $NVRM^i = VRM^i$ because the one of the input dimensions is positive. Since two of the output dimensions are non-positive for Case 1, we have $VRM^o = 1 - \gamma^*$. As shown in Table 4, for Case 1, the value of $VRM^o = 1 - \gamma^*$ ranges from 0.4496 to 1 whereas the value of VRM^o in Table 2 ranges from 0.645 to 1.

For Case 2, the input dimension is negative. As shown in Table 3, the some of the values for VRM^i are negative because those β^* values are greater than 1. For example, the VRM^i for DMU 1 is -3.0014 whose β^* value is 4.0014. According the definition of $NVRM^i$, we have $NVRM^i = \dfrac{1}{1+\beta^*} > 0$. As shown in Table 4, for Case 2, the input-oriented efficiencies are positive under the new measure $NVRM^i$. For Case 2, the values of VRM^o in Table 3 range from 0.59 to 1, which are limited in the range between 0.5 and 1, whereas the values for

Table 4. Efficiencies for new VRM efficiency measure.

DMU	Case 1: 2 Inputs		Case 2: 1 Inputs	
	$NVRM^i$	$NVRM^o$	$NVRM^i$	$NVRM^o$
1	0.9417	0.8962	0.1999	0.5030
2	0.6812	0.7005	0.3884	0.6178
3	1.0000	1.0000	1.0000	1.0000
4	0.5580	0.5374	0.1525	0.5374
5	0.8628	0.7023	0.1011	0.3051
6	0.8584	0.8390	0.1430	0.4596
7	1.0000	1.0000	0.7249	0.7361
8	1.0000	1.0000	1.0000	1.0000
9	0.8113	0.9039	0.5000	0.9039
10	0.4540	0.6307	0.2983	0.6159
11	1.0000	1.0000	1.0000	1.0000
12	0.5101	0.4496	0.3189	0.3850
13	1.0000	1.0000	1.0000	1.0000

$NVRM^o$ in Table 4 range from 0.3051 to 1, which indicates the range of $NVRM^o$ is wider than the range of VRM^o.

V. CONCLUSION

This study has investigated the properties of the efficiency measures for VRM proposed by Cheng et al. (2013). We have developed the new efficiency measures for VRM in both input-oriented and output-oriented models. Our measure assures that the efficiency in input-oriented model is positive and the efficiency in output-oriented model will not be limited to the range [0.5, 1]. The properties which we identified provide more inside information in defining the efficiency measure with directional distance functions.

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