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EFFICIENCY MEASUREMENT FOR INTERNATIONAL CONTAINER PORTS OF TAIWAN AND SURROUNDING AREAS BY FUZZY DATA ENVELOPMENT ANALYSIS

Yu-Jie Wang and Tzeu-Chen Han

Key words: fuzzy data envelopment analysis (fuzzy DEA), global logistics, international container ports.

ABSTRACT

International container ports are critical hubs for global logistics, and high efficiency is a particularly important requirement for managing these ports. International container ports in Taiwan and surrounding areas have to deal with increasingly larger amounts of cargo in recent years. As a result, it has become more difficult to measure these ports' efficiency. This study offers a new approach for measuring the efficiency of international ports. Twelve international container ports in Taiwan and surrounding areas were chosen as the samples, and their six inputs (application service process, service personnel ability, service personnel attitude, advisory services, harbor rates and stevedoring rates), and seven outputs (tug boat operation, rope untwisting operation, pilot operation, stevedoring efficiency, low damage rate for goods, awaiting unloading and working, and service flexibility) are analyzed herein with fuzzy data envelopment analysis (DEA). This approach allows objective and easy measurement of international container port efficiency.

I. INTRODUCTION

Transportation modes of transnational transportation vary from port to port, making international container ports (Wang, 2016) a critical consideration in global logistics, for which these ports operate as hubs (Chou, 2010). Since container lines constructed according to new specifications have increased cargo loadings, international container ports must now deal with constantly increasing amounts of cargo. Based on the above, highly efficient international ports in transnational transportation significantly influences the business development of the country owning the ports and their surrounding areas. Furthermore, high port efficiency promotes economic growth, while low port efficiency may slow economic growth. In short, the measurement of international port efficiency is an important issue because the efficiency of these ports significantly affects the economic growth of a country, especially Taiwan and surrounding areas.

In Taiwan and surrounding areas, important international container ports include: Kaohsiung, Hong Kong, Shenzhen, Shanghai, Qingdao, Pusan, Tokyo, Manila, Tanjung Priok, Singapore, Klang and Laem Chabang. In the past, numerous criteria (Wang, 2016) were considered in some approaches to evaluate the performance of international container ports, and the work was viewed as a multi-criteria decision-making (MCDM) problem (Hwang and Yoon, 1981; Chang et al., 2008). Additionally, linguistic terms were employed by experts based on objective standards to express criteria ratings, and individual opinions to present the weights of criteria. The linguistic ratings (Delgado et al., 1992; Herrera et al., 1996) and weights were then converted into fuzzy numbers (Zadeh, 1965; Zimmermann, 1987; Zimmermann, 1991). Since the criteria weight was judged subjectively, the evaluation problem in past approaches was recognized as a fuzzy multi-criteria decision-making (FMCDM) problem (Wang and Lee, 2010; Wang, 2014). Moreover, the subjective judgment of the criteria weights in FMCDM could be unreasonable and controversial because experts often have different opinions concerning weights. To resolve the problem, this study classifies criteria for international container ports of Taiwan and surrounding areas (Wang, 2016) into inputs and outputs, and the criteria weights are not taken into consideration. The efficiency of the international container ports of Taiwan and surrounding areas is then measured by fuzzy data envelopment analysis (fuzzy DEA) (Puo and Tanaka, 2001; Angiz et al., 2012).

Fuzzy DEA is an extension of data envelopment analysis (DEA) (Charnes et al., 1978). DEA is a technique used to measure relative efficiency values of peer decision-making units (DMUs) with some inputs and outputs under their uncertain corresponding multipliers. Practically, international port efficiency measurement items, including some inputs and outputs, are easily

Paper submitted 05/03/17; revised 08/17/17; accepted 09/28/17. Author for correspondence: Tzeu-Chen Han (Email: tchan@gms.npu.edu.tw).
Department of Shipping and Transportation Management, National Penghu University of Science and Technology, Penghu, Taiwan, R.O.C.

assessed by experts according to their professional experience, whereas it is unnecessary to determine the corresponding weights of items in DEA. Since DEA was proposed by Charnes et al. (1978), numerous DEA approaches have been applied in various methodologies and applications (Lee et al., 2011; Hwang et al., 2013). In DEA, input and output weights of decision-making units (DMUs) can be decided by themselves. Comparing DEA with MCDM, DEA DMUs are viewed as alternatives of MCDM (Lee et al., 2014).

Previously, Chou et al. (2004) used cross-time recursive DEA (RDEA) to evaluate container port efficiency in China and Taiwan. Due to the specific data (i.e., fuzzy inputs and outputs), Chou et al.'s cross-time RDEA, used with crisp values, is not adequate or suitable for the problem of evaluating ports under fuzzy environment. Based on fuzzy inputs and outputs, the proposed computation in this paper will be fuzzy DEA. This study utilizes fuzzy DEA to measure the efficiency of international container ports in Taiwan and surrounding areas. Furthermore, in the future, RDEA, including cross-time RDEA, may also be extended in fuzzy environments to solve related evaluation problems through the proposed fuzzy DEA. For the sake of clarity, mathematical preliminaries, including fuzzy numbers and DEA, are expressed in Section 2. In Section 3, a fuzzy DEA is improved from the traditional DEA (Charnes et al., 1978) for measuring the efficiency of international container ports of Taiwan and surrounding areas. An empirical study concerning the issue is illustrated in Section 4. Finally, conclusions are presented in Section 5.

II. MATHEMATICAL PRELIMINARIES

In this section, the related notions of fuzzy numbers (Zadeh, 1965; Zimmermann, 1987; Zimmermann, 1991) and DEA (Charnes et al., 1978) are presented.

Definition 2.1 Let U be a universe set. A fuzzy subset A of U is defined by a membership function $\mu_A(x) \to [0, 1]$, where $\mu_A(x)$, $\forall x \in U$ denotes the degree of x in A. Commonly, the function $\mu_A(x)$ is the generalization of characteristic function for a crisp subset. The fuzzy set A of U is characterized by a membership function with the value x representing "degree of membership" of x in A. Thus the fuzzy set A is defined by $A = \{(x, \mu_A(x)) | x \in U\}$ or $\int_{x\in U} \mu_A(x)/x$.

Definition 2.2 The $\alpha - cut$ of fuzzy set A is a crisp set $A^\alpha = \{x | \mu_A(x) \ge \alpha\}$ and the support A is the crisp set $Supp(A) = \{x | \mu_A(x) > 0\}$.

Definition 2.3 A fuzzy subset A of U is normal iff $\sup_{x\in U} \mu_A(x) = 1$.

Definition 2.4 A fuzzy subset A of U is convex iff $\mu_A(\lambda x + (1-\lambda)y) \ge (\mu_A(x) \wedge \mu_A(y))$, $\forall x, y \in U$, $\forall \lambda \in [0, 1]$, where $\wedge$ indicates the minimum operator.

Definition 2.5 A fuzzy subset A of U is a fuzzy number if A is both normal and convex.

Definition 2.6 A triangular fuzzy number A is a fuzzy set with piecewise linear membership function μ_A defined by:

$$\mu_A = \begin{cases} \dfrac{x - a^L}{a^M - a^L} & if \;\; a^L \le x \le a^M \\ \dfrac{a^U - x}{a^U - a^M} & if \;\; a^M \le x \le a^U \\ 0, & otherwise \end{cases},$$

which can be denoted as a triplet (a^L, a^M, a^U).

Definition 2.7 Let the i^{th} input of DMU_j be indicated as x_{ij} and the r^{th} output of DMU_j be denoted as y_{rj}, where $i = 1, 2, \ldots, m$; $r = 1, 2, \ldots, s$; $j = 1, 2, \ldots, n$. Charnes et al. (1978) proposed a well-known DEA model called the CCR model with input orientation for DMU_k, presented as a multiplier form as follows:

$$\text{Max } \theta_k = \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}}$$

$$\text{s.t. } \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \quad j = 1, 2, \ldots, n,$$

$$u_r, v_i \ge \varepsilon > 0 \, ; \; i = 1, 2, \ldots, m \, ; \; r = 1, 2, \ldots, s \, .$$

Additionally, ε is a non-Archimedean small positive number. Due to its fractional programming form, solving the above model under input orientation is difficult. Thus Charnes et al. (1978) transformed the fractional programming model into a linear programming model expressed in the following:

$$\text{Max} \sum_{r=1}^{s} u_r y_{rk}$$

$$\text{s.t. } \sum_{i=1}^{m} v_i x_{ik} = 1,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad j = 1, 2, \ldots, n,$$

$$u_r, v_i \ge \varepsilon > 0 \, ; \; i = 1, 2, \ldots, m \, ; \; r = 1, 2, \ldots, s \, .$$

Definition 2.8 Let the i^{th} input of DMU_j be denoted as x_{ij} and the r^{th} output of DMU_j be indicated as y_{rj}, where $i = 1, 2, \ldots, m$; $r = 1, 2, \ldots, s$; $j = 1, 2, \ldots, n$. Charnes et al. (1978) proposed a famous DEA model called the CCR model with output orientation for DMU_k shown as a multiplier form:

$$\text{Min } \beta_k = \frac{\sum_{i=1}^{m} v_i x_{ik}}{\sum_{r=1}^{s} u_r y_{rk}}$$

$$\text{s.t. } \frac{\sum_{i=1}^{m} v_i x_{ij}}{\sum_{r=1}^{s} u_r y_{rj}} \geq 1 \quad j = 1, 2, \ldots, n,$$

$$u_r, v_i \geq \varepsilon > 0 \;;\; i = 1, 2, \ldots, m \;;\; r = 1, 2, \ldots, s .$$

Like the DEA model described in Definition 2.7, the fractional programming model can be transferred into a linear programming form presented as:

$$\text{Min } \sum_{i=1}^{m} v_i x_{ik}$$

$$\text{s.t. } \sum_{r=1}^{s} u_r y_{rk} = 1 ,$$

$$\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \geq 0 \quad j = 1, 2, \ldots, n ,$$

$$u_r, v_i \geq \varepsilon > 0 \;;\; i = 1, 2, \ldots, m \;;\; r = 1, 2, \ldots, s .$$

In CCR under input orientation and output orientation, it is obvious that $\theta_k^* = 1/\beta_k^*$ according to the same restrictions of the DEA models described in Definitions 2.7 and 2.8, where θ_k^* and β_k^* respectively represent the optimal solutions of objective functions θ_k and β_k.

Based on the above definitions, this study utilizes fuzzy numbers and DEA to extend a fuzzy DEA for efficiency measurement of international container ports in Taiwan and surrounding areas.

III. FUZZY DEA FOR EFFICIENCY MEASUREMENT OF INTERNATIONAL CONTAINER PORTS

In the past, Wang and Lee (2010) applied an FMCDM method with weakness and strength indices to evaluate financial performance of Taiwan container shipping companies. Then Wang (2016) also utilized the same FMCDM method to evaluate performance of international container ports in Taiwan and surrounding areas. Wang evaluated the performance of corresponding problems with the FMCDM method. Obviously, the FMCDM method is very useful. With the above approaches, experts had to assess ratings for the different alternatives of all criteria, and even determine the weights of those criteria. Practically, assessing ratings is easy and objective based on some evaluation standards, while determining the weights of criteria is hard and subjective due to varying expert opinions. In order to avoid this problem, this study proposes a fuzzy DEA for efficiency measurement of international container ports without criterion weights. Herein, alternatives are viewed as DMUs and criteria are divided into inputs and outputs. By eliminating the need for criterion weights, the fuzzy DEA easily and objectively measures the efficiency of international container ports in Taiwan and surrounding areas. In this study, twelve international container ports, namely: Kaohsiung, Hong Kong, Shenzhen, Shanghai, Qingdao, Pusan, Tokyo, Manila, Tanjung Priok, Singapore, Klang and Laem Chabang are respectively denoted as DMU1, DMU2, …, DMU12. In order to measure the efficiency of each of these international container ports, data on four port items are obtained: administration, fees, harbor service and handling service. Of these port items, administration and fees are input items, and harbor service and handling service are output items. Additionally, thirteen inputs and outputs that fall under the above four related port items are:

(1) Administration:
Administration items include four inputs: application service process, service personnel ability, service personnel attitude and advisory services. The four inputs are important because excellent administration items will improve port execution.

(2) Fees:
Fee items directly impact a port's cost and benefit. To measure the efficiency of international container ports, related fees include two inputs: harbor rates and stevedoring rates. Harbor rates represent corresponding port working expenses per unit, while stevedoring rates express corresponding port operating expenses per unit.

(3) Harbor service:
Harbor service items focus on the harbor operation of container lines, which includes three outputs: tug boat operation, rope untwisting operation and pilot operation. The above operations have an impact on crews' working performance in port. Thus, high harbor service can result in high harbor expenses.

(4) Handling service:
Handling service items focus on ship handling works, which include four outputs: stevedoring efficiency, low damage rate for goods, awaiting unloading and working, and service flexibility. The service directly influences ship working efficiency in port, so high handling service can require high handling expenses.

In short, inputs based on administration and fees are respec-

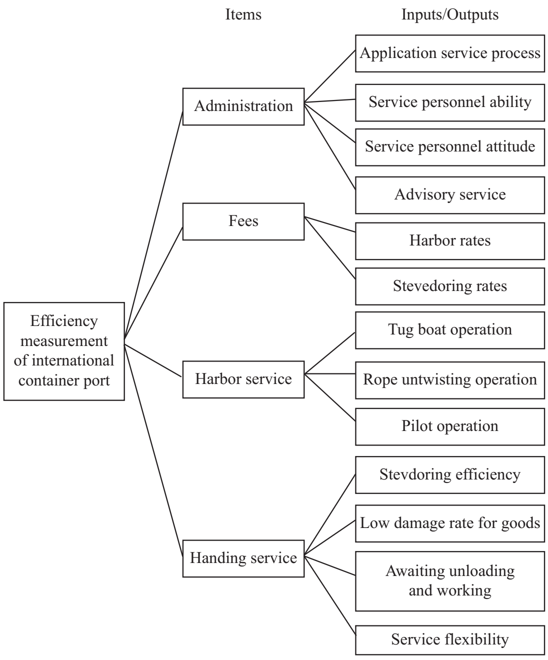

Fig. 1. The relationship between four items and thirteen inputs/outputs.

tively application service process, service personnel ability, service personnel attitude, advisory services, harbor rates, and stevedoring rates, denoted as x1, x2, …, x6, while outputs based on harbor service and handling service are respectively tug boat operation, rope untwisting operation, pilot operation, stevedoring efficiency, low damage rate for goods, awaiting unloading and working, and service flexibility, denoted as y1, y2, …, y7. Additionally, the relationships between the four items and the thirteen inputs/outputs are shown in Fig. 1.

The inputs and outputs are assessed by linguistic terms (Delgado et al., 1992; Herrera et al., 1996), and then converted into fuzzy numbers (Zadeh, 1965; Zimmermann, 1987; Zimmermann, 1991). The two models described in Definitions 2.7 and 2.8 are used, in which DMUs are precisely presented, whereas fuzzy DEA is a powerful tool for measuring the efficiency of DMUs with imprecise data (Puo and Tanaka, 2001; Angiz et al., 2012). Considering fuzzy inputs and outputs, the model described in Definition 2.7 can be naturally extended to the following fuzzy DEA model (Puo and Tanaka, 2001; Angiz et al., 2012).

Definition 3.1 Let the i^{th} fuzzy input of DMU_j be denoted as $\tilde{x}_{ij}$ and the r^{th} output of DMU_j be denoted as $\tilde{y}_{rj}$, where $i = 1, 2, \ldots, m$; $r = 1, 2, \ldots, s$; $j = 1, 2, \ldots, n$. The fuzzy DEA model for measuring DMU_k efficiency under input orientation is expressed as follows:

$$\max \tilde{\theta}_k = \frac{\sum_{r=1}^{s} u_r \tilde{y}_{rk}}{\sum_{i=1}^{m} v_i \tilde{x}_{ik}}$$

$$\text{s.t.} \ \frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}} \tilde{\leq} \tilde{1}, \ j = 1, 2, \ldots, n,$$

$$u_r, v_i \geq \varepsilon > 0; \ i = 1, 2, \ldots, m; \ r = 1, 2, \ldots, s,$$

where "~" denotes the fuzziness, and thus $\tilde{\leq}$ and $\tilde{1}$ respectively indicate the fuzziness of ≤ and 1.

Additionally, the fuzzy DEA model above is transformed into a linear programming form presented as:

$$\max \sum_{r=1}^{s} u_r \tilde{y}_{rk}$$

$$\text{s.t.} \ \sum_{i=1}^{m} v_i \tilde{x}_{ik} = \tilde{1},$$

$$\sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \tilde{\leq} \tilde{0}, \ j = 1, 2, \ldots, n,$$

$$u_r, v_i \geq \varepsilon > 0; \ i = 1, 2, \ldots, m; \ r = 1, 2, \ldots, s,$$

where $\tilde{0}$ represents the fuzziness of 0.

Definition 3.2 Let the i^{th} fuzzy input of DMU_j be denoted as $\tilde{x}_{ij}$ and the r^{th} output of DMU_j be denoted as $\tilde{y}_{rj}$, where $i = 1, 2, \ldots, m$; $r = 1, 2, \ldots, s$; $j = 1, 2, \ldots, n$. The fuzzy DEA model under output orientation for measuring DMU efficiency is expressed as:

$$\min \tilde{\beta}_k = \frac{\sum_{i=1}^{m} v_i \tilde{x}_{ik}}{\sum_{r=1}^{s} u_r \tilde{y}_{rk}}$$

$$\text{s.t.} \ \frac{\sum_{i=1}^{m} v_i \tilde{x}_{ij}}{\sum_{r=1}^{s} u_r \tilde{y}_{rj}} \tilde{\geq} 1, \ j = 1, 2, \ldots, n,$$

$$u_r, v_i \geq \varepsilon > 0; \ i = 1, 2, \ldots, m; \ r = 1, 2, \ldots, s,$$

where $\tilde{\geq}$ denotes the fuzziness of ≥.

Similarly, the fuzzy DEA model under input orientation can be transferred into a linear programming form:

$$\min \sum_{i=1}^{m} v_i \tilde{x}_{ik}$$

$$\text{s.t.} \sum_{r=1}^{s} u_r \tilde{y}_{rk} = \tilde{1},$$

$$\sum_{i=1}^{m} v_i \tilde{x}_{ij} - \sum_{r=1}^{s} u_r \tilde{y}_{rj} \tilde{\geq} 0, \; j = 1, 2, \ldots, n,$$

$$u_r, v_i \geq \varepsilon > 0; \; i = 1, 2, \ldots, m; \; r = 1, 2, \ldots, s.$$

The two above models are merely a conceptual description because the related computations in fuzzy numbers are difficult in practice. Therefore, this study proposes fuzzy DEA according to CCR and fuzzy number characteristics. Let $\tilde{x}_{ij}$ be represented by three characteristic values $(x_{ij}^L, x_{ij}^M, x_{ij}^U)$ and $\tilde{y}_{rj}$ be expressed by three characteristic values $(y_{rj}^L, y_{rj}^M, y_{rj}^U)$, where x_{ij}^L is a value in the left boundary of $\tilde{x}_{ij}$; x_{ij}^M represents a value in $\tilde{x}_{ij}$ among which its membership function value equals 1; x_{ij}^U is a value in the right boundary of $\tilde{x}_{ij}$; y_{rj}^L is the left boundary of $\tilde{y}_{rj}$; y_{rj}^M represents a value in $\tilde{y}_{rj}$ among which its membership function value equals 1; and y_{rj}^U is a value in the right boundary of $\tilde{y}_{rj}$. Obviously, $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$ are triangular fuzzy numbers as they have piecewise linear membership functions, where $i = 1, 2, \ldots, m$; $r = 1, 2, \ldots, s$; $j = 1, 2, \ldots, n$.

From the above description, let the i^{th} fuzzy input of DMU_j be denoted as $(x_{ij}^L, x_{ij}^M, x_{ij}^U)$ and the r^{th} output of DMU_j be denoted as $(y_{rj}^L, y_{rj}^M, y_{rj}^U)$ for $i = 1, 2, \ldots, m$; $r = 1, 2, \ldots, s$; $j = 1, 2, \ldots, n$. The fuzzy DEA model under input orientation for measuring DMU_k efficiency is then expressed as:

$$\max w\theta_k^L + g\theta_k^M + b\theta_k^U = \frac{\sum_{r=1}^{s} u_r (wy_{rk}^L + gy_{rk}^M + by_{rk}^U)}{\sum_{i=1}^{m} v_i (wx_{ik}^U + gx_{ik}^M + bx_{ik}^L)}$$

$$\text{s.t.} \frac{\sum_{r=1}^{s} u_r (wy_{rj}^L + gy_{rj}^M + by_{rj}^U)}{\sum_{i=1}^{m} v_i (wx_{ij}^U + gx_{ij}^M + bx_{ij}^L)} \leq 1 \quad j = 1, 2, \ldots, n,$$

$$u_r, v_i \geq \varepsilon > 0; i = 1, 2, \ldots, m; r = 1, 2, \ldots, s;$$

$$w, g, b \in \{0, 1\} \text{ and } w + g + b = 1.$$

Likewise, the fuzzy DEA model above can be transformed into a linear programming form:

$$\max w\theta_k'^L + g\theta_k'^M + b\theta_k'^U = \sum_{r=1}^{s} u_r (wy_{rk}^L + gy_{rk}^M + by_{rk}^U)$$

$$\text{s.t.} \sum_{i=1}^{m} v_i (wx_{ik}^U + gx_{ik}^M + bx_{ik}^L) = 1,$$

$$\sum_{r=1}^{s} u_r (wy_{rj}^L + gy_{rj}^M + by_{rj}^U) - \sum_{i=1}^{m} v_i (wx_{ij}^U + gx_{ij}^M + bx_{ij}^L) \leq 0$$

$$j = 1, 2, \ldots, n,$$

$$u_r, v_i \geq \varepsilon > 0; i = 1, 2, \ldots, m; r = 1, 2, \ldots, s;$$

$$w, g, b \in \{0, 1\} \text{ and } w + g + b = 1.$$

For DMU_k, $\theta_k'^{L*}$ is the optimal objective function value in the worst situation as $w = 1$; $\theta_k'^{M*}$ is the optimal objective function value in a general situation as $g = 1$; and $\theta_k'^{U*}$ is the optimal objective function value in the best situation as $b = 1$.

Based on $\theta_k'^{L*}$, $\theta_k'^{M*}$ and $\theta_k'^{U*}$, the fuzzy efficiency value of DMU_k under input orientation is derived as:

$$(E_k^L, E_k^M, E_k^U), \text{ where } E_k^L = \min\{\theta_k'^{L*}, \theta_k'^{M*}, \theta_k'^{U*}\},$$

$$E_k^M = \text{median}\{\theta_k'^{L*}, \theta_k'^{M*}, \theta_k'^{U*}\}, \text{ and}$$

$$E_k^U = \max\{\theta_k'^{L*}, \theta_k'^{M*}, \theta_k'^{U*}\}.$$

In traditional DEA models such as CCR, the optimal efficiency value in objective function for a DMU is equal to 1, which indicates that the DMU under input orientation is efficient. However, the DMU under input orientation is inefficient as its efficiency value is smaller than 1. Based on the above, the efficiency of DMU_k in the fuzzy DEA model under input orientation is determined by its fuzzy efficiency value (E_k^L, E_k^M, E_k^U). The judgment rules under input orientation are presented below:

(1) Inefficient:
A DMU under input orientation is considered inefficient if $E_k^U < 1$.

(2) Slightly efficient:
A DMU under input orientation is considered slightly efficient as $E_k^L < 1$ and $E_k^M < 1$, but $E_k^U = 1$.

(3) Partially efficient:
A DMU under input orientation is considered partially efficient as $E_k^M = 1$ and $E_k^U = 1$, but $E_k^L < 1$.

(4) Efficient:
A DMU under input orientation is considered efficient if $E_k^L = 1$.

On the other hand, a fuzzy DEA model under output orientation for measuring DMU_k efficiency is expressed as:

$$\min w\beta_k^L + g\beta_k^M + b\beta_k^U = \frac{\sum_{i=1}^{m} v_i(wx_{ik}^U + gx_{ik}^M + bx_{ik}^L)}{\sum_{r=1}^{s} u_r(wy_{rk}^L + gy_{rk}^M + by_{rk}^U)}$$

$$\text{s.t.} \quad \frac{\sum_{i=1}^{m} v_i(wx_{ij}^U + gx_{ij}^M + bx_{ij}^L)}{\sum_{r=1}^{s} u_r(wy_{rj}^L + gy_{rj}^M + by_{rj}^U)} \geq 1 \quad j = 1, 2, \ldots, n,$$

$$u_r, v_i \geq \varepsilon > 0; i = 1, 2, \ldots, m; r = 1, 2, \ldots, s;$$

$$w, g, b \in \{0, 1\} \text{ and } w + g + b = 1.$$

The fuzzy DEA model above can be transferred into a linear programming form:

$$\min w\beta_k'^L + g\beta_k'^M + b\beta_k'^U = \sum_{i=1}^{m} v_i(wx_{ik}^U + gx_{ik}^M + bx_{ik}^L)$$

$$\text{s.t.} \quad \sum_{r=1}^{s} u_r(wy_{rk}^L + gy_{rk}^M + by_{rk}^U) = 1,$$

$$\sum_{i=1}^{m} v_i(wx_{ij}^U + gx_{ij}^M + bx_{ij}^L) - \sum_{r=1}^{s} u_r(wy_{rj}^L + gy_{rj}^M + by_{rj}^U) \geq 0$$

$$j = 1, 2, \ldots, n,$$

$$u_r, v_i \geq \varepsilon > 0; i = 1, 2, \ldots, m; r = 1, 2, \ldots, s;$$

$$w, g, b \in \{0, 1\} \text{ and } w + g + b = 1.$$

For DMU_k, $\beta_k'^{L*}$ is the optimal objective function value in the worst situation as $w = 1$; $\beta_k'^{M*}$ is the optimal objective function value in a general situation as $g = 1$; and $\beta_k'^{U*}$ is the optimal objective function value in the best situation as $b = 1$.

Based on $\beta_k'^{L*}$, $\beta_k'^{M*}$ and $\beta_k'^{U*}$, the fuzzy efficiency value of DMU_k under output orientation is yielded as $(E_k'^L, E_k'^M, E_k'^U)$, where $E_k'^L = \min\{\beta_k'^{L*}, \beta_k'^{M*}, \beta_k'^{U*}\}$, $E_k'^M = \text{median}\{\beta_k'^{L*}, \beta_k'^{M*}, \beta_k'^{U*}\}$, and

$$E_k'^U = \max\{\beta_k'^{L*}, \beta_k'^{M*}, \beta_k'^{U*}\}.$$

In traditional DEA models such as CCR, the optimal efficiency value in the objective function for a DMU under output orientation is equal to 1, which indicates that the DMU under output orientation is efficient, whereas the DMU under output orientation is inefficient as its efficiency value is greater than 1.

Based on the above, the efficiency of DMU_k in a fuzzy DEA model under output orientation is determined by its fuzzy efficiency value $(E_k'^L, E_k'^M, E_k'^U)$. The judgment rules under output orientation are presented as follows:

(1) Inefficient:
A DMU under output orientation is considered inefficient if $E_k'^L > 1$.
(2) Slightly efficient:
A DMU under output orientation is considered slightly efficient as $E_k'^M > 1$ and $E_k'^U > 1$, but $E_k^L = 1$.
(3) Partially efficient:
A DMU under output orientation is considered partially efficient as $E_k'^L = 1$ and $E_k'^M = 1$, but $E_k'^U > 1$.
(4) Efficient:
A DMU under output orientation is considered efficient if $E_k'^U = 1$.

Using these rules, it is possible to determine the efficiency of DMUs under input and output orientations. In traditional DEA, DMUs measured are merely designated as either efficient or inefficient. However, the fuzzy DEA proposed in this study classifies DMUs with fuzzy inputs into four varied situations: inefficient, slightly efficient, partially efficient and efficient, respectively, due to the concepts of fuzzy numbers and peer DMUs. For instance, the evaluation "slightly efficient" indicates that DMU is measured as being only efficient in the worst situation, a general situation, or the best situation. In addition, the evaluation "partially efficient" indicates that DMU is measured as being inefficient in the worst situation, a general situation, or the best situation.

In fact, the proposed fuzzy DEA differs from Kao and Liu's (2000) fuzzy efficiency measure in DEA in that their method relies completely on an extension principle of fuzzy sets (Zadeh, 1965; Zimmermann, 1987; Zimmermann, 1991). Thus, their model could only offer representations in which DMU was measured as being in the best situation, while the other references were in the worst situation; or in which DMU was measured as being in the worst situation while the other references were in the best situation. On the other hand, the proposed fuzzy DEA considers fuzzy numbers and peer DMUs in synchrony. That is to say, both an evaluated DMU and the other references based on the characteristic of fuzzy numbers will be simultaneously in the worst situation, a general situation, or the best situation to emphasize and enhance DMUs' peer characteristic.

IV. EMPIRICAL STUDY

In order to demonstrate the efficiency measurement for international container ports of Taiwan and surrounding areas clearly, this study conducted an empirical study based on data obtained from 59 questionnaires chosen from 75. Of the 59 chosen questionnaires, the efficiency values of the twelve international container ports DMU1, DMU2, …, DMU12 were measured using the six fuzzy inputs x1, x2, …, x6 and seven fuzzy outputs y1,

Table 1. The fuzzy inputs and outputs of international container ports.

	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10	DMU11	DMU12
x1	(2, 3.86, 5)	(2, 4.31, 5)	(2, 3.75, 5)	(2, 3.59, 5)	(2, 3.66, 5)	(3, 3.86, 5)	(1, 4.39, 5)	(1, 2.86, 5)	(2, 3.27, 5)	(3, 4.14, 5)	(2, 3.58, 5)	(2, 3.39, 5)
x2	(2, 3.90, 5)	(2, 4.25, 5)	(2, 3.71, 5)	(2, 3.61, 5)	(2, 3.53, 5)	(2, 3.81, 5)	(3, 4.41, 5)	(1, 2.71, 5)	(2, 3.07, 5)	(3, 4.08, 5)	(2, 3.47, 5)	(2, 3.34, 5)
x3	(2, 3.85, 5)	(3, 4.24, 5)	(2, 3.54, 5)	(2, 3.53, 5)	(2, 3.42, 5)	(2, 3.69, 5)	(3, 4.29, 5)	(1, 2.63, 5)	(1, 2.90, 4)	(2, 4.19, 5)	(2, 3.34, 5)	(2, 3.15, 5)
x4	(3, 3.78, 5)	(3, 4.31, 5)	(2, 3.63, 5)	(2, 3.64, 5)	(2, 3.46, 5)	(2, 3.76, 5)	(3, 4.32, 5)	(1, 2.59, 5)	(1, 2.88, 5)	(2, 4.22, 5)	(2, 3.37, 5)	(2, 3.24, 5)
x5	(2, 3.73, 5)	(1, 3.53, 5)	(2, 3.63, 5)	(2, 3.49, 5)	(2, 3.59, 5)	(2, 3.63, 5)	(1, 3.41, 5)	(1, 3.03, 5)	(2, 3.19, 4)	(2, 3.63, 5)	(2, 3.49, 5)	(2, 3.39, 5)
x6	(2, 3.78, 5)	(1, 3.51, 5)	(2, 3.66, 5)	(2, 3.59, 5)	(2, 3.59, 5)	(2, 3.63, 5)	(1, 3.32, 5)	(1, 3.02, 5)	(2, 3.27, 5)	(2, 3.66, 5)	(2, 3.54, 5)	(2, 3.41, 5)
y1	(2, 4.03, 5)	(3, 4.47, 5)	(3, 3.85, 5)	(2, 3.64, 5)	(2, 3.81, 5)	(2, 3.88, 5)	(1, 4.25, 5)	(1, 3.03, 5)	(2, 3.32, 5)	(1, 4.07, 5)	(1, 3.54, 5)	(2, 3.46, 5)
y2	(2, 3.92, 5)	(2, 4.29, 5)	(2, 3.80, 5)	(2, 3.42, 5)	(2, 3.81, 5)	(2, 3.73, 5)	(2, 4.19, 5)	(1, 3.05, 5)	(1, 3.17, 5)	(1, 3.93, 5)	(2, 3.49, 5)	(2, 3.39, 5)
y3	(1, 3.78, 5)	(3, 4.37, 5)	(2, 3.83, 5)	(1, 3.46, 5)	(2, 3.78, 5)	(3, 3.86, 5)	(3, 4.44, 5)	(1, 2.98, 5)	(1, 3.15, 5)	(2, 4.05, 5)	(1, 3.46, 5)	(2, 3.46, 5)
y4	(2, 3.97, 5)	(3, 4.49, 5)	(3, 4.10, 5)	(2, 4.12, 5)	(3, 3.90, 5)	(2, 4.02, 5)	(3, 4.47, 5)	(1, 2.98, 5)	(2, 3.20, 5)	(3, 4.36, 5)	(2, 3.61, 5)	(2, 3.49, 5)
y5	(3, 3.97, 5)	(3, 4.34, 5)	(3, 3.93, 5)	(2, 3.78, 5)	(2, 3.75, 5)	(3, 3.97, 5)	(3, 4.42, 5)	(1, 2.81, 5)	(2, 3.10, 5)	(3, 4.20, 5)	(1, 3.39, 5)	(2, 3.39, 5)
y6	(2, 4.05, 5)	(3, 4.47, 5)	(2, 4.03, 5)	(2, 4.00, 5)	(3, 3.81, 5)	(2, 3.80, 5)	(3, 4.46, 5)	(1, 2.83, 5)	(1, 3.00, 4)	(2, 4.24, 5)	(2, 3.34, 4)	(2, 3.44, 5)
y7	(2, 3.93, 5)	(3, 4.24, 5)	(2, 3.88, 5)	(2, 3.80, 5)	(1, 3.75, 5)	(2, 3.75, 5)	(3, 4.47, 5)	(1, 2.78, 5)	(2, 3.19, 5)	(2, 4.08, 5)	(2, 3.39, 5)	(2, 3.42, 5)

y2, …, y7, shown in Fig. 1. As in Wang's (2016) approach, linguistic terms ("very bad (VB)", "bad (B)", "medium (M)", "good (G)", "very good (VG)") based on the Likert scale concept (Cooper and Schindler, 2014) were respectively converted into "very bad (VB) = 1", "bad (B) = 2", "medium (M) = 3", "good (G) = 4" and "very good (VG) = 5" to evaluate the thirteen inputs and outputs. 59 experts' linguistic opinions were aggregated into fuzzy numbers for inputs and outputs by the converting formulas as follows:

Let X_{ij}^k be a linguistic opinion employed by the k^{th} expert E_k for DMU_j against the i^{th} input, and the linguistic opinion be represented by a corresponding crisp value x_{ij}^k, where $i = 1, 2, \ldots, 6$; $j = 1, 2, \ldots, 12$; $k = 1, 2, \ldots, 59$. Then $\tilde{x}_{ij}$ indicates the fuzzy opinion of DMU_j on the i^{th} input, where $i = 1, 2, \ldots, 6$; $j = 1, 2, \ldots, 12$.

Let

$$\tilde{x}_{ij} = (x_{ij}'^{L}, x_{ij}'^{M}, x_{ij}'^{U})$$

where

$$x_{ij}'^{L} = \min_k \{x_{ij}^k\},$$

$$x_{ij}'^{M} = \sum_k x_{ij}^k / 59,$$

$$x_{ij}'^{U} = \max_k \{x_{ij}^k\}$$

for

$$i = 1, 2, \ldots, 6;\ j = 1, 2, \ldots, 12.$$

Similarly, let Y_{rj}^k be a linguistic opinion employed by the k^{th} expert E_k for DMU_j against the r^{th} output, and the linguistic opinion be represented by a corresponding crisp value y_{rj}^k, where $r = 1, 2, \ldots, 7$; $j = 1, 2, \ldots, 12$; $k = 1, 2, \ldots, 59$. Then, $\tilde{y}_{rj}$ indicates the fuzzy opinion of DMU_j on the r^{th} output, where $r = 1, 2, \ldots, 7$; $j = 1, 2, \ldots, 12$.

Let

$$\tilde{y}_{rj} = (y_{rj}'^{L}, y_{rj}'^{M}, y_{rj}'^{U})$$

where

$$y_{rj}'^{L} = \min_k \{y_{rj}^k\},$$

$$y_{rj}'^{M} = \sum_k y_{rj}^k / 59,$$

$$y_{rj}'^{U} = \max_k \{y_{rj}^k\}$$

$$\text{for } r = 1, 2, \ldots, 7;\ j = 1, 2, \ldots, 12.$$

Based on the above computations, the six fuzzy inputs and seven outputs to measure the efficiency of the twelve international container ports are expressed in Table 1. According to the converting formulas, the left boundary of each fuzzy number is derived by the minimum value of all opinions, and the right boundary of each fuzzy number is yielded by the maximum value of all opinions. Thus the left boundary and right boundary of all fuzzy numbers are presented by integers, whereas the middle value of each fuzzy number derived by mean computation may not be an integer.

However, there is a measuring tie in Table 1 because the number of DMUs is less than the summary of the number of inputs and outputs. According to rule of thumb, the number of DMUs should not be less than double the summary of the number of inputs and outputs. Fortunately, these fuzzy inputs and outputs can be merged along varied items in Fig. 1 to resolve the measuring tie as follows:

Table 2. The fuzzy inputs and outputs of international container ports after merged.

	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10	DMU11	DMU12
X1	(9, 15.39, 20)	(10, 17.11, 20)	(8, 14.63, 20)	(8, 14.37, 20)	(8, 14.07, 20)	(9, 15.12, 20)	(10, 17.41, 20)	(4, 10.79, 20)	(6, 12.12, 19)	(10, 16.63, 20)	(8, 13.76, 20)	(8, 13.12, 20)
X2	(4, 7.51, 10)	(2, 7.04, 10)	(4, 7.29, 10)	(4, 7.08, 10)	(4, 7.18, 10)	(4, 7.26, 10)	(2, 6.73, 10)	(2, 6.05, 10)	(4, 6.46, 9)	(4, 7.29, 10)	(4, 7.03, 10)	(4, 6.8, 10)
Y1	(5, 11.73, 15)	(8, 13.13, 15)	(7, 11.48, 15)	(5, 10.52, 15)	(6, 11.4, 15)	(7, 11.47, 15)	(6, 12.88, 15)	(3, 9.06, 15)	(4, 9.64, 15)	(4, 12.05, 15)	(4, 10.49, 15)	(6, 10.31, 15)
Y2	(9, 15.92, 20)	(12, 17.54, 20)	(10, 15.94, 20)	(8, 15.7, 20)	(9, 15.21, 20)	(9, 15.54, 20)	(12, 17.82, 20)	(4, 11.4, 20)	(7, 12.49, 19)	(10, 16.88, 20)	(7, 13.73, 19)	(8, 13.74, 20)

Table 3. The fuzzy efficiency evaluations derived by the fuzzy DEA model under input orientation for international container ports in Taiwan and surrounding areas.

DMU_j	θ'^{L*}_j	θ'^{M*}_j	θ'^{U*}_j	(E^L_j, E^M_j, E^U_j)	Judgment
DMU1	0.75	0.962	0.5	(0.5, 0.75, 0.962)	Inefficient
DMU2	1	1	1	(1, 1, 1)	Efficient
DMU3	0.875	1	0.5	(0.5, 0.875, 1)	Slightly efficient
DMU4	0.667	1	0.5	(0.5, 0.667, 1)	Slightly efficient
DMU5	0.75	1	0.5	(0.5, 0.75, 1)	Slightly efficient
DMU6	0.875	0.961	0.5	(0.5, 0.875, 0.961)	Inefficient
DMU7	1	1	1	(1, 1, 1)	Efficient
DMU8	0.375	1	1	(0.375, 1, 1)	Partially efficient
DMU9	0.648	0.965	0.667	(0.648, 0.667, 0.965)	Inefficient
DMU10	0.833	0.962	0.5	(0.5, 0.833, 0.962)	Inefficient
DMU11	0.583	0.938	0.5	(0.5, 0.583, 0.938)	Inefficient
DMU12	0.75	0.969	0.5	(0.5, 0.75, 0.969)	Inefficient

Based on the concept, $X_{ij} = (x^L_{ij}, x^M_{ij}, x^U_{ij})$ for $i = 1, 2; j = 1, 2, \ldots, 12$ and $Y_{rj} = (y^L_{rj}, y^M_{rj}, y^U_{rj})$ for $r = 1, 2; j = 1, 2, \ldots, 12$ are summarized from corresponding items and yielded as:

$$X_{1j} = \sum_{i=1}^{4} \tilde{x}_{ij},\ X_{2j} = \sum_{i=5}^{6} \tilde{x}_{ij},\ Y_{1j} = \sum_{r=1}^{3} \tilde{y}_{rj}, \text{ and } Y_{2j} = \sum_{r=4}^{7} \tilde{y}_{rj}.$$

Then the two inputs and two outputs are presented in Table 2. Based on Table 2, the fuzzy efficiency evaluations derived by the fuzzy DEA model under input orientation for international container ports are expressed in Table 3, where ε is assumed to be 0.000001.

Table 3 shows that DMU1, DMU6, DMU9, DMU10, DMU11 and DMU12 were inefficient, DMU3, DMU4 and DMU5 were slightly efficient, DMU8 was partially efficient, and DMU2 and DMU7 were efficient. DMU2 and DMU7 were clearly more efficient than the other DMUs. As described in the previous section, DMUs measured by fuzzy DEA are classified as inefficient, slightly efficient, partially efficient or efficient due to the characteristics of fuzzy numbers. On the other hand, DMUs measured by traditional DEA are classified as either inefficient or efficient. Therefore, using fuzzy DEA in DMU measurement provides more information for decision-making than using traditional DEA.

Additionally, the fuzzy efficiency situations derived by the fuzzy DEA model under output orientation based on Table 2 for international container ports are expressed in Table 4, where ε is assumed to be 0.000001. The obtained result is consistent with that of Table 3.

Practically, (E'^L_j, E'^M_j, E'^U_j) for DMU_j under output orientation can be omitted to simplify yielding fuzzy DMU efficiency values, where $j = 1, 2, \ldots, 12$. According to the description of CCR (i.e., $\theta^*_k = 1/\beta^*_k$) in Section 2, the efficiency values under input orientation and output orientation are related. Furthermore, $(E^L_j, E^M_j, E^U_j) \approx (1/E'^U_j, 1/E'^M_j, 1/E'^L_j)$, and thus the judgment result in Table 3 is the same as the judgment result in Table 4.

V. CONCLUSIONS

This study has extended traditional DEA, such as CCR, into fuzzy DEA using fuzzy number characteristics in order to measure the efficiency of twelve international container ports in Taiwan and surrounding areas without having to consider weighting values of inputs and outputs. In fuzzy DEA, twelve international container ports in Taiwan and surrounding areas were chosen as DMUs, and linguistic ratings on input and output items for the DMUs were converted into fuzzy numbers. Based on the characteristics of fuzzy numbers, this study applied fuzzy DEA to solve the efficiency measurement problem for the twelve international container ports without the need to assign or consider weights of inputs and outputs. The proposed fuzzy DEA consists of three components: the worst situation, a general situation, and the best situation. Following the three situations, fuzzy DEA

Table 4. The fuzzy efficiency evaluations derived by the fuzzy DEA model under output orientation for international container ports in Taiwan and surrounding areas.

DMU_j	$\beta_j'^{L*}$	$\beta_j'^{M*}$	$\beta_j'^{U*}$	(E_j^L, E_j^M, E_j^U)	Judgment
DMU1	1.333	1.039	2	(1.039, 1.333, 2)	Inefficient
DMU2	1	1	1	(1, 1, 1)	Efficient
DMU3	1.143	1	2	(1, 1.143, 2)	Slightly efficient
DMU4	1.5	1	2	(1, 1.5, 2)	Slightly efficient
DMU5	1.333	1	2	(1, 1.333, 2)	Slightly efficient
DMU6	1.143	1.041	2	(1.041, 1.143, 2)	Inefficient
DMU7	1	1	1	(1, 1, 1)	Efficient
DMU8	2.667	1	1	(1, 1, 2.667)	Partially efficient
DMU9	1.543	1.036	1.5	(1.036, 1.5, 1.543)	Inefficient
DMU10	1.2	1.039	2	(1.039, 1.2, 2)	Inefficient
DMU11	1.714	1.066	2	(1.066, 1.714, 2)	Inefficient
DMU12	1.333	1.032	2	(1.032, 1.333, 2)	Inefficient

under input orientation and output orientation can yield fuzzy efficiency values for DMUs respectively. By the fuzzy DEA computation, it was found that the results of judging under input orientation and output orientation were consistent. The consistent results are useful because the fuzzy DEA computation can be omitted in execution under input orientation or output orientation. Practically, this is because the fuzzy DEA is a fuzzy extension of CCR, and CCR characteristics produce the same result. The simplifying computation is easy and rational for measuring the efficiency of international container ports. Another advantage of measuring DMUs based on the characteristics of fuzzy numbers with fuzzy DEA is that more information can be gathered and represented than when using traditional DEA in crisp values.

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