SHAPE OPTIMIZATION OF RECTANGULAR MULTI-CHAMBER MUFFLERS AT HIGH-ORDER-MODES

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Key words: eigen function, genetic algorithm, higher order mode, optimization.

ABSTRACT

Muffler design as of late has been restricted to lower frequencies using the plane wave theory. This has led to an underestimation of acoustical performances at higher frequencies. To overcome the above shortcomings, an analysis of three-dimensional waves propagating for a simple muffler using the finite element method has been developed. However, there has been scant research on rectangular mufflers equipped with baffle plates that eliminate noise, nor has the space-constrained conditions of industrial muffler designs been properly addressed. Therefore, to improve the acoustical performance of a rectangular muffler within a constrained space, the shape optimization of rectangular mufflers using an eigen function, a four-pole system in conjunction with a Genetic Algorithm is proposed and assessed. Here, a numerical analysis of pure tone noise emitted from an induced fan using three kinds of rectangular mufflers hybridized with multiple chambers is introduced. Before the optimization process is carried out, an accuracy check of the mathematical models for two rectangular one-chamber/three-chamber mufflers with various allocations of inlets/outlets and partitioned bafflers is also performed. Simulated results reveal that the acoustical performance of a muffler with more geometric design parameter is found to be more efficient. And finally, the STL of the muffler will increase as the number of the rectangular chambers increases.

I. INTRODUCTION

The researches of mufflers have been widely addressed. Munjal (1975) developed the advanced four-pole transfer matrix using the fluid dynamic theory. Sullivan and Crocker (1978, 1979) presented coupled equations for the outer and inner tubes of a perforated muffler. Jayaraman and Yam (1981) developed an analytic solution for the coupled equations. However, all the solutions obtained were based on the plane wave theory which neglected the acoustical effect of a higher order wave. To overcome this weakness, Ih and Lee (1985, 1897) introduced the acoustical performance of an expansion muffler with a circular section. Later, Venkatesham et al. (2009) analyzed the acoustical performance of a rectangular expansion chamber with arbitrary location of inlet/outlet using Green’s functions. Abom (1990) deduced a four-pole matrix for an extended muffler with a circular section. Munjal (1987) simplified the calculation process using a numerical analysis method. This method can be easily used in the expansion muffler with rectangular and circular sections; however, the ratio of the expansion area to the inlet/outlet area should be an integer value. Additionally, it is difficult to analyze a muffler using the analytic method if the angle between inlet and outlet is 90 degrees. Because more calculation time is needed, this type of muffler is often analyzed using the finite element method (Young and Crocker, 1975) or the two-dimensional boundary element method (Seybert and Cheng, 1987). Ih (1992) developed a numerical method to analyze the acoustical performance of the expansion muffler hybridized with a circular/rectangular section and an inlet/outlet duct. The inlet/outlet ducts may be parallel with/without the same height, or they may be perpendicular to each other. Considering the orthogonality of modes in each chamber, Selamet and Ji (1998) developed a mathematical mode of circular flow-reversing chambers. To find the appropriate design parameters within a space-constrained situation, Yeh et al. (2004) and Chang et al. (2005) assessed shape optimizations of multi-segment and double-chamber mufflers using the plane wave theory. Yeh et al. (2006) analyzed and optimized a space-constrained reactive muffler in low frequencies in 2006. Similarly, Chiu and Chang (2008), developed multi-chamber perforated mufflers and explored the optimal muffler shapes under plane wave situation. Later, Chiu (2010a; 2010b; 2010c), proposed three kinds of mufflers (multi-chamber mufflers equipped with plug-inlet tube, one-chamber muffler equipped with reverse-flow ducts, and three-chamber mufflers hybridized with a side inlet and a perforated tube) and evaluated the best shape of the muffler. And, Chang et al. (2010) developed opti-
mally shaped one-chamber mufflers equipped with perforated intruding tubes using a SA Method. Later, Chiu and Chang (2011a, 2011b, 2013) presented three kinds of optimally shaped mufflers (two-chamber mufflers with perforated plug/non-plug tubes, multi-chamber perforated mufflers, and multi-chamber side inlet/outlet mufflers hybridized with multiple perforated intruding tubes). Subsequently, Chiu (2013) presented one-chamber perforated mufflers filled with wool and explored its optimal shape. However, the effect of higher order wave was ignored. So, considering the high-order-mode wave effect, Chiu and Chang (2014) investigated the acoustical performance of mufflers which has one-chamber and inlet and outlet. Mimani and Munjal (2016a, 2016b), developed an optimal design on reactive rectangular expansion chambers and flow-reversal end-chamber mufflers. Because of simple shape and few chamber (one chamber) design for the muffler, the noise abatement in eliminating high noise is insufficient; therefore, in order to enhance the acoustical performance of the muffler, three kinds of complicated mufflers hybridized with multiple rectangular chambers (muffler A: a stacked three-chamber muffler; muffler B: a parallel three-chamber muffler; muffler C: a stacked four-chamber muffler) in conjunction with a GA method are proposed.

II. THEORETICAL BACKGROUND

Concerning the higher order wave, the mathematical models of two rectangular one-chamber mufflers with different allocations of inlets and outlets shown in Fig. 1 will be deduced. In order to enhance the acoustical performance of the muffler, three kinds of rectangular mufflers (muffler A: a stacked three-chamber muffler; muffler B: a parallel three-chamber muffler; muffler C: a stacked four-chamber muffler) with multiple chambers shown in Fig. 2 are proposed. Based on the analysis of the higher order modes, the acoustical performances of rectangular mufflers A-C will be established. Finally, the objective functions of the three mufflers at a target tone will be constructed.

1. Derivation of Mathematical Models

The sound wave for one-chamber rectangular mufflers is shown in Fig. 1. As derived by Ih (1992) and Venkatesham et al. (2009), when the inlet and outlet are on the same side or opposite side, the acoustical pressure at inlet (p_11) and outlet (p_22) are expressed as

$$ p_{11} = \left( -1 \right)^{j} j U_{1} Z_{q} \left\{ \frac{1}{\tan kl} \left( \frac{b}{h_{1} h_{2}} \right) \left[ \sum \sum \left( \frac{2}{m \pi} \right) \left( \frac{h_{1}}{h} \right)^{2} \psi_{111}^{2} \right] \right. $$

$$ + \sum \sum \left( \frac{2}{m \pi} \right) \left( \frac{h_{1}}{h} \right)^{2} \psi_{112}^{2} \right. $$

$$ \left. + \sum \sum \left( \frac{4}{m n \pi} \right) \left( \frac{2}{m n \pi} \right) \psi_{111}^{2} \psi_{112}^{2} \right\} \left[ k \tan kl \right] $$

$$ = \left( -1 \right)^{j} j U_{1} Z_{q} E_{11} $$

(1a)
For a one-chamber rectangular muffler having the inlet and outlet on the same side (Fig. 1(a)), using the superposition method at inlet and outlet and rearranging \( P \) and \( U \), the acoustical transfer matrix between nodes 1 and 2 is

\[
\begin{bmatrix}
P_1 \\ U_1
\end{bmatrix} =
\begin{bmatrix}
TS_{1,1} & TS_{1,2} \\ TS_{2,1} & TS_{2,2}
\end{bmatrix}
\begin{bmatrix}
P_2 \\ U_2
\end{bmatrix}
\tag{4a}
\]

(1b)

Where

\[
TS_{1,1} = \left( \frac{P_1}{P_2} \right)|_{\eta_1=0} = E_{11} / E_{12},
\]

\[
TS_{1,2} = \left( \frac{P_1}{U_2} \right)|_{\eta_1=0} = jZ_0 (E_{12} - E_{11}E_{22} / E_{12}),
\]

\[
TS_{2,1} = \left( U_1 / P_2 \right)|_{\eta_1=0} = j(Z_0E_{12})^{-1},
\]

\[
TS_{2,2} = \left( U_1 / U_2 \right)|_{\eta_1=0} = E_{22} / E_{12}.
\]

Similarly, a one-chamber rectangular muffler having the inlet and outlet on the opposite side (Fig. 1(b)), the acoustical transfer matrix between nodes 1 and 2 is

\[
\begin{bmatrix}
P_1 \\ U_1
\end{bmatrix} =
\begin{bmatrix}
TR_{1,1} & TR_{1,2} \\ TR_{2,1} & TR_{2,2}
\end{bmatrix}
\begin{bmatrix}
P_2 \\ U_2
\end{bmatrix}
\tag{5}
\]

Based on the above derivation, the acoustical performances of mufflers A-C shown in Fig. 2 will be established as follows:

For muffler A, the system’s four-pole matrix and STL are

\[
\begin{bmatrix}
P_1 \\ U_1
\end{bmatrix} =
\begin{bmatrix}
TS_{1,1} & TS_{1,2} \\ TS_{2,1} & TS_{2,2}
\end{bmatrix}
\begin{bmatrix}
P_2 \\ U_2
\end{bmatrix}
\tag{6a}
\]

\[
STL_{s}(f, \bar{X}_1) = 20 \log \left[ \frac{T_{f_{11}} + T_{f_{12}} + T_{21}Z_1 + T_{22}(S_1 / S_2)}{2} \right]
\tag{6b}
\]

Where

\[
\bar{X}_1 = (L_1, L_2, B_01, B_02, H_02)
\tag{6c}
\]

Similarly, for muffler B, the system’s four-pole matrix and STL are

\[
\begin{bmatrix}
P_1 \\ U_1
\end{bmatrix} =
\begin{bmatrix}
\Pi_{n=1} T_n(f) \\ U_4
\end{bmatrix}
\begin{bmatrix}
P_4 \\ U_4
\end{bmatrix}
\tag{7a}
\]

Where

\[
\Pi_{n=1} T_n(f) = T_{f_{11}} + T_{f_{12}} + T_{21}Z_1 + T_{22}(S_1 / S_2)
\tag{7b}
\]
where \( m \) is the number of the chambers,

\[
\vec{x}_2 = (B_{01}, B_{02}, H_{02}), B_{01} = B_{03},
\]

\[
H_{01} = H_{03} = H_x,
\]

\[
L_1 = L_2 = L_3
\]

Likewise, for muffler C, the system’s four-pole matrix and STL are

\[
\begin{bmatrix}
P_1 \\
U_1
\end{bmatrix} = \prod_{m=1}^{4} \begin{bmatrix} T_m(f) \\
1
\end{bmatrix},
\begin{bmatrix}
P_3 \\
U_5
\end{bmatrix} = \begin{bmatrix} T_{T11} & T_{T12} \\
T_{T21} & T_{T22}
\end{bmatrix},
\begin{bmatrix}
P_5 \\
U_5
\end{bmatrix}
\]

\[
STL_3(f, \vec{x}_3) = 20 \log \left[ \frac{T_{T11} + \frac{T_{T12}}{Z_2} + T_{T21}Z_1 + T_{T22}(S_y / S_t)}{2} \right]
\]

where \( m \) is the number of the chambers,

\[
\vec{x}_3 = (L_0, B_0, H_0), L_1 = L_2
\]

2. Objective Function

By using the formulas of Eqs. (6)-(8), the objective functions used in the GA optimization with respect to each type of plenum are established. For muffler A, the objective function in maximizing the STL at one tone \( (f) \) is

\[
OBJ_1 = STL_1(f_1, \vec{x}_1)
\]

Similarly, for muffler B, the objective function in maximizing the STL at one tone \( (f) \) is

\[
OBJ_2 = STL_2(f_1, \vec{x}_2)
\]

Also, for muffler C, the objective function in maximizing the STL at one tone \( (f) \) is

\[
OBJ_3 = STL_3(f_1, \vec{x}_3)
\]

III. MODEL CHECK

Before performing the GA optimal simulation on rectangular mufflers, an accuracy check of the mathematical models on the mufflers (a one-chamber concentric reverse muffler and a three-chamber concentric straight muffler) is performed using the analytic solution from Ih (2009). As depicted in Figs. 3 and 4, the theoretical prediction from Eqs. (1)-(5) and Ih’s analytic data are in agreement. Therefore, the proposed fundamental mathematical models for the multi-chamber rectangular mufflers with inlets and outlets on the same side or opposite side are acceptable. Consequently, the models linked with the numerical method are applied to the shape optimization of rectangular mufflers A-C shown in Fig. 2 in the following section.

Obtaining the best acoustical performance by adjusting the design parameters, numerical assessments linked to a GA optimizer are applied. The corresponding OBJ functions and ranges of the design parameters are summarized in Eqs. (9)-(11) and Table 1.

IV. CASE STUDY

In this paper, an induced fan installed inside a machine room is used to convey the air from the machine to the atmosphere via a building’s venting duct. In order to depress noise energy (I.D. fan) from the I.D. fan’s outlet, three kinds of multi-chamber mufflers (mufflers A-C shown in Fig. 2) installed within a constrained space is proposed. Considering current geometric status, muffler A is designed as a stacked three-chamber muffler, muffler B is designed as a parallel three-chamber muffler, and muffler C
is planned as a stacked four-chamber muffler. Here, muffler B
is attached onto the beam with a width of Lx and a height of Hx.
Muffler C is attached onto the beam with a width of Bx and a
height of Hx. The original sound power level (SWLO) has been
investigated to be remarkable at 500 Hz. Here, the pure tone
of 500 Hz is the fan’s blade passing frequency (Bf) which has
been obtained with
\[ B_f = \frac{N_b \times \text{rpm}}{60} \]
where \( N_b \) is the number of fan’s blade and rpm is the revolution(s) per minute for the
driven motor.

### V. GENETIC ALGORITHM

Traditional gradient methods (such as EPFM, IPFM and FDM)
need a good starting point before the optimization process is
performed. However, the genetic algorithm (GA) does not need
an appropriate starting point. Therefore, GA becomes easier
to use. In addition, the GA’s accuracy can be improved when all
the GA parameters of crossover, mutation and elitism are taken
into consideration (Chang et al., 2005). Here, the GA method
used to search for the global optimum by imitating a genetic evo-
lutionary process was formalized by Holland (1975) and extended
to functional optimization by Jong (1975). It has been widely
used in various fields (Yeh et al., 2004; Chang et al., 2005; Yeh
et al., 2006; Chiu and Chang, 2008; Chiu, 2010; Chiu and Chang,
2014). For the GA optimization, design parameters of \( X_1, X_2, \ldots, X_k \) were determined by searching for a better objective func-
tion \( OBJ \).

Presetting the bit (the bit length of the chromosome) and the pop (the population number), the interval of the \( k \)-th design parameter \( X_k \) with \( [Lb_k, Ub_k] \) was then mapped to the band
of the binary value. The mapping system between the variable interval of \( [Lb, Ub] \) and the \( k \)-th binary chromosome of

was then established. The encoding from \( x \) to \( B2D \) (binary to
decimal) can be performed as

\[
B2D_k = \text{integer} \left( \frac{x_k - Lb_k}{Ub_k - Lb_k} \left( 2^{bi} - 1 \right) \right)
\]  

(12)

By evaluating the \( OBJ \), the whole set of chromosomes \( [B2D_1,
B2D_2, \ldots, B2D_k] \) that changed from binary form to decimal
form was then assigned a fitness by decoding the transformation
system.

\[
\text{fitness} = OBJ(X_1, X_2, \ldots, X_k);
\]  

(13)

where

\[
X_k = B2D_k \ast (Ub_k - Lb_k)/(2^{bi} - 1) + Lb_k
\]

The GA operation is shown in Fig. 5. In order to assess an eli-
tism of a gene, the tournament selection, a random comparison
of the relative fitness from pairs of chromosomes, was adopted.
In addition, one pair of offspring was generated from the selected
parent via uniform crossover with a probability of \( pc \). A mutation
occurred with a probability of \( pm \) where the new and unexpected
point was brought into the GA optimizer’s search domain.

As indicated in Fig. 6, the process was terminated when the
number of generations reached a pre-selected value of \( iter_{\text{max}} \).

### VI. RESULTS AND DISCUSSION

#### 1. Results

The accuracy of the GA optimization depends on five types
of GA parameters including \( pop \) (population number), \( bit \)
(chromosome length), $pc$ (crossover rate), $pm$ (mutation factor), and $iter_{max}$ (maximum iteration). To achieve a good optimization, the following parameters are varied step by step. The related GA control parameters used in the optimization are shown in Table 2.

![Diagram](image)

**Fig. 5. Operations of the GA method.**

**Fig. 6. Flow diagram of the GA method.**

<table>
<thead>
<tr>
<th>GA parameters</th>
<th>Value (or condition)</th>
</tr>
</thead>
<tbody>
<tr>
<td>bit</td>
<td>20</td>
</tr>
<tr>
<td>pop</td>
<td>50</td>
</tr>
<tr>
<td>elitism</td>
<td>(tournament)</td>
</tr>
<tr>
<td>crossover</td>
<td>(uniform crossover)</td>
</tr>
<tr>
<td>$pc$</td>
<td>0.5</td>
</tr>
<tr>
<td>$pm$</td>
<td>0.5</td>
</tr>
<tr>
<td>$iter_{max}$</td>
<td>1000</td>
</tr>
</tbody>
</table>

**Table 2. Selected GA parameters during shape optimization.**

Utilizing Eqs. (9)-(11) and using the GA parameters in the optimization process, the maximization of the $STL_1$ to $STL_3$ with respect to mufflers A-C at targeted tones (500 Hz, 1000 Hz, 2000 Hz) was performed and shown in Tables 3-5. As illustrated in Table 3, the STL of mufflers A-C at 500 Hz has been improved from 34 dB, 36 dB, 45 dB to 110 dB, 47 dB, and 125 dB. As can be seen in Table 4, the STL of mufflers A-C at 1000 Hz has been improved from 68 dB, 25 dB, and 100 dB to 115 dB, 70 dB, and 194 dB. Similarly, as depicted in Table 5, the STL of mufflers A-C at 2000 Hz has been improved from 30 dB, 27 dB, and 56 dB to 85 dB, 43 dB, and 140 dB.

Using this optimal design in a theoretical calculation, the com-
Table 3. Comparison of STL and design parameters of muffler A before and after the optimization (target tone of 500 Hz).

<table>
<thead>
<tr>
<th>Target Tone</th>
<th>Design Parameters</th>
<th>OBJ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L1 (m)</td>
<td>B01 (m)</td>
</tr>
<tr>
<td>Muffler A</td>
<td>original</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>optimization</td>
<td>0.2032</td>
</tr>
</tbody>
</table>

Table 4. Comparison of STL and design parameters of muffler A before and after the optimization (target tone of 1000 Hz).

<table>
<thead>
<tr>
<th>Target Tone</th>
<th>Design Parameters</th>
<th>OBJ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L1 (m)</td>
<td>B01 (m)</td>
</tr>
<tr>
<td>Muffler A</td>
<td>original</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>optimization</td>
<td>0.3250</td>
</tr>
</tbody>
</table>

Table 5. Comparison of STL and design parameters of muffler A before and after the optimization (target tone of 2000 Hz).

<table>
<thead>
<tr>
<th>Target Tone</th>
<th>Design Parameters</th>
<th>OBJ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L1 (m)</td>
<td>B01 (m)</td>
</tr>
<tr>
<td>Muffler A</td>
<td>original</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>optimization</td>
<td>0.3734</td>
</tr>
</tbody>
</table>

Comparison of STL curves with respect to mufflers A-C before and after optimization at targeted tones (500 Hz, 1000 Hz, 2000 Hz) are plotted in Figs. 7-9, Figs. 10-12, and Fig. 13-15. Consequently, the optimal STL curves with respect to mufflers A-C at targeted tone (500 Hz) is plotted in Fig. 16.

2. Discussion

As can be seen in Figs. 7-9, Figs. 10-12, and Fig. 13-15, the
STLs of the mufflers have been precisely maximized at the targeted tones. As indicated in Table 3, the STL with respect to mufflers A to C at 500 Hz has been improved by 76 dB, 11 dB, and 80 dB. Similarly, as depicted in Table 4, the STL with respect to mufflers A to C at 1000 Hz has been improved by 47 dB, 45 dB, and 94 dB. Likewise, as illustrated in Table 5, the STL...
with respect to mufflers A to C at 2000 Hz has been improved by 55 dB, 16 dB, and 84 dB. As indicated in Table 3 and Fig. 16, the STL with respect to mufflers A-C at target tone (500 Hz) reaches 110 dB, 47 dB, and 125 dB. Likewise, as indicated in Table 4, the STL with respect to mufflers A-C at target tone (1000 Hz) reaches 115 dB, 70 dB, and 194 dB. Also, as illustrated in Table 5, the STL with respect to mufflers A-C at target tone (2000 Hz) is 85 dB, 43 dB, and 140 dB. Obviously, muffler C (a four-chamber muffler composed of two reverse chambers and two straight chambers) has the best acoustical performance. Moreover, the STL of muffler A (a three-chamber muffler composed of two straight chambers and a reverse chamber) is better than that of muffler B (a three-chamber muffler composed of three reverse chambers). Here, considering the geometric limit (structure beam) inside the building, the number of muffler A’s geometrical design parameter available to use to reach optimal STL by adjusting the muffler shape is four. However, the number of muffler B’s available geometrical design parameters is three only. With this, the acoustical performance of B is worse than that of muffler A. In addition, more chambers in the muffler (such as muffler C) will result in a better acoustical performance.

VII. CONCLUSION

It has been shown that the optimization of muffler shapes can be easily and efficiently carried out by using a four-pole transfer matrix as well as a GA optimizer. Considering the noise elimination of a pure tone noise propagating along the venting duct installed inside the building, three kinds of mufflers with rectangular section are adopted and optimized using GA algorithm. Optimal results in Figs. 7-15 indicate that the predicted maximal value of the STL can be located at the desired frequencies (500, 1000, and 2000 Hz). Hence, the tuning ability established by adjusting the design parameters of mufflers A-C in conjunction with GA method is assured. Moreover, as indicated in Figs. 7-16, the STL profile of muffler C (a four-chamber muffler composed of two reverse chambers and two straight chambers) is better than that of muffler A (a three-chamber muffler composed of two straight chambers and a reverse chamber) and
muffler B (a three-chamber muffler composed of three reverse chambers).

Consequently, results reveal that more number of muffler’s geometrical design parameters used in optimization process will obtain better acoustical performance. In addition, more chambers used in the muffler will result in a better acoustical performance.

**VIII. NOMENCLATURE**

This paper is constructed on the basis of the following notations:

- $c_0$: sound speed (m/s)
- $f$: frequency (Hz)
- $\alpha$: angular frequency ($= 2\pi f$) (rad/s)
- $\rho_0$: air density (kg/m$^3$)
- $m, n$: acoustical mode
- $k$: wave number of sound wave $\left(\frac{\omega}{c_0}\right)$
- $k_x$: wave number of sound wave in x-axis $\left(\frac{m\pi}{b}\right)$
- $k_y$: wave number of sound wave in y-axis $\left(\frac{n\pi}{h}\right)$
- $k_z$: wave number of sound wave in z-axis $\sqrt{k_x^2 - k_y^2 - k_z^2}$
- $b$: width of expansion chamber (m)
- $h$: height of expansion chamber (m)
- $l$: length width of expansion chamber (m)
- $b_i$: width of inlet (m)
- $h_i$: height of inlet (m)
- $b_o$: width of outlet (m)
- $h_o$: height of outlet (m)
- $(b_{i1}, h_{i1})$: center coordinate of inlet (m)
- $(b_{o2}, h_{o2})$: center coordinate of outlet (m)
- $S_e$: section area of the expansion chamber (m$^2$)
- $S_i$: section area of inlet (m$^2$)
- $S_o$: section area of outlet (m$^2$)
- $U_\omega$: acoustical volume velocity (m$^3$/s)
- $Z_e$: acoustical impedance of the expansion chamber in z-axis (raysl/m)
- $Z_i$: acoustical impedance of inlet in z-axis (raysl/m)
- $Z_o$: acoustical impedance of outlet in z-axis (raysl/m)
- $\bar{P}_a$: averaged acoustical pressure (Pa)
- $\bar{P}_{ac}$: averaged interactive acoustical pressure (Pa)
- $\bar{P}_1$: acoustical potential energy in the inlet (joule)
- $\bar{P}_2$: acoustical potential energy in the outlet (joule)

**ACKNOWLEDGEMENTS**

The authors acknowledge the financial support of the National Science Council (MOST 104-2221-E-036-027, R.O.C.).

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