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FAST EIGENVALUE ANALYSIS OF CRITICAL OSCILLATION MODE FOR SHIPBOARD ELECTRIC POWER SYSTEM

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Key words: shipboard electric power system, small signal stability, eigenvalue, damping of oscillation, critical mode.

ABSTRACT

Frequency domain analysis has been widely adopted for the study of the small signal stability of the power system. The frequency domain approach includes the linearization of the system to obtain a linear model as well as the system matrix of which the eigenvalues can be calculated to determine the system stability. However, we often have high order of system matrix and thus it will be undesirable to calculate and analyze all the system eigenvalues.

This paper is to explore the problem of small signal stability for the shipboard electric power system and the main purpose is to find out the worst-damped mode of system eigenvalues and thus to alleviate the effort for computing and analyzing all the system eigenvalues. A sample shipboard power system is taken as the study system. The worst-damped mode of system eigenvalues under different operating conditions are calculated for fast analysis and comparative investigations of the system small signal stability.

I. INTRODUCTION

The primary function of an electric power system is to provide users with economic and reliable electric power with high quality. In accordance with the different purposes of utilization, electric power systems can be divided into several categories including residential, commercial, and industrial power systems, as well as those systems for transportation vehicles such as railways, ships, and aircrafts.

The most important task for electric power system operation is to maintain system stability under various possible operating points, including continuous variations of the load. The stability of electric power system is usually divided into three types, namely: steady-state stability, transient stability, and small signal stability. The steady-state stability generally refers to the maximum electric power transmission capacity of the electric power transmission line. The transient stability is based on the consideration of whether or not the system can be kept at a stable operation under major disturbances. The small signal stability concerns the dynamic responses when the system is subjected to small disturbances. It is noted that the stability margin of the small signal stability tends to be more stringent than that of the transient stability (Anderson and Fouad, 1994; Kundur, 1994; Rogers, 2000).

The shipboard electric power system is the kind of electric power system with independent operating characteristics of which the various loads on board are in constant and continuous changes and the operating circumstances are far severer when the ship is navigating on the sea. Regarding both personnel safety and power supply reliability, the requirements for design and maintenance of the shipboard electric power system are much more stringent than those for the ordinary on-shore electric power systems.

In general the analysis of the small signal stability of the power system is based on frequency domain analysis method by which system state equations will be derived and the eigenvalues of system state matrix will be calculated with respect to the operating point in order to determine whether this system is stable or not. Ordinary on-shore power systems usually consist of a large number of generators, and the major dynamic behaviors of this system are mainly from the generators. With the capacity of each customer being far less than the capacity of the generator, the dynamic effect of single load can be neglected in the mathematical model of the overall system. On the other hand, the shipboard electric power system is a kind of small-scale independent system. Although the number of generators in a shipboard electric power system is rather small, the system dynamic behaviors, such as voltage and frequency, are vulnerable to load variations. There are a large number of working loads installed on the ship and they are mostly electric motors of different capacities. Each motor load consumes a certain percentage of the system total load and thus the dynamic effect of the load shall not be neglected in the mathematical model of the overall system. Therefore, the stability study of the on-shore as well as the shipboard electric power system will have to face

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the system state matrix with rather high orders. It will take a very long time to calculate all eigenvalues with respect to every operating point, and there will be too many eigenvalues which may be quite difficult to analyze.

Some literatures have proposed rapid calculation method with respect to small signal stability analysis on electric power system state matrix with high orders. In 1995 Campagnolo et al. proposed various methods such as Parallel LSSI Algorithm, Parallel BI Algorithm and Hybrid Parallel Algorithm (Campagnolo et al., 1995) for calculating several critical eigenvalues near the specific shift point. In 1995 Lima et al. proposed a new method for small signal stability analysis (Lima et al., 1995). This method is a new matrix transformation suitable for small signal stability analysis of a large power system, and it is based on a calculation of several critical eigenvalues near the specific shift point by using Power Method, inverse iteration method, and S-matrix method. In all aforementioned methods, only the critical eigenvalues near the shift point have been taken into consideration such that it cannot immediately determine whether the system is stable. If the most unstable (or with the lowest damping) eigenvalue is nowhere near the shift point, it will lead to misjudgment of system stability. Until now, the literature related to the calculation of partial eigenvalues is the calculation of critical eigenvalues by Jacobi-Davidson QR proposed by Tsai et al. in 2010 (Tsai et al., 2010). In this paper, the real number type of Jacobi-Davidson numerical algorithm in conjunction with a strategy of the flexible selection of critical eigenvalue has been utilized to solve the critical eigenvalues of large power system state matrix.

In this paper a method for rapid determination of system stability is proposed to solve the most unstable (or with the lowest damping) and all unstable eigenvalues, and to further solve the corresponding eigenvectors with high accuracy in order to avoid time consumption for calculating all eigenvalues. A sample shipboard electric power system is used to test and verify the proposed algorithm in order to ascertain that it can indeed achieve the rapid analysis of the small signal stability of the power system.

II. SHIPBOARD ELECTRIC POWER SYSTEM

The applications of electric power systems can be rather diversified. An electric power system can be as large as an ordinary ground electric power system, or as small as the electric power systems for ships, railroads, and factories, or even the circuits that supply electric power to batteries. The electric power systems can be divided into two major categories: interconnected electric power system and independent electric power system. The shipboard electric power system itself is an independent power supply system. The operational environment of the shipboard electric power system is very different from that of the ground electric power system so that it is equipped with various unique operation features which can be quite different from those of a ground electric power system.

The shipboard electric power system itself is a system with independent operation features and various kinds of equipment and different operation requirements such that it has more frequent variations of operating states. With the more severe working environment and requirements of higher electric power quality for precision equipment on the ship, the design and maintenance of the shipboard electric power system should follow a more stringent standard than the ground electric power system based on the premises of assuring personnel safety and the electric power quality. With more stringent operating conditions of the shipboard electric power system, the stability analysis of the system at each operation condition has become even more important.

The composition of the shipboard electric power system is identical to the ground electric power system which includes power generating units, power delivery network, and electricity load on the ship.

The major characteristics of the shipboard electric power system are as follows: (1) the electric power is completely supplied by the self-equipped power generation system, (2) as a small electric power system, the total power generation capacity of this system is smaller than the general electric power system on the ground, (3) with a single load accounting for larger ratio of total capacity, the variation of the operation condition of this kind of load (such as activation or shut-down) will have greater impact of the system, (4) the operation condition of the load changes more frequently, (5) the work environment of electric equipment is worse leading to higher likelihood of accidents, and (6) requiring a wider variety of equipment with more special equipment control functionalities (Smith, 1983; Lee, 1990; Watson, 1990; McGeorge, 1993; IEEE Standard 45-1998, 1998). From these characteristics we can realize that the operation conditions of the shipboard electric power system is a series of continuous dynamic processes. Therefore, applying the small signal stability analysis to such systems with less reoccuring operating points is rather important.

III. SMALL SIGNAL STABILITY ANALYSIS

Power system small signal stability is often referred to as power system dynamic stability and it focuses on the ability of the system to maintain stable subject to small disturbances (Anderson and Fouad, 1994; Kundur, 1994; Rogers, 2000). Instead of employing the time domain approach of applying various small disturbances on the system to observe the dynamic behaviors of the system, the frequency domain approach, i.e., performing eigenanalysis by calculating the eigenvalue/eigenvector of the system matrix of the linearized system under study, has been widely adopted in the industry for power system small signal stability analysis. Eigen-analysis is primarily based on modal expansion theory (modal analysis).

Consider the linear unforced system described in (1):

$$\dot{x}(t) = \mathbf{A}x(t),\ x(0) = x_0 \tag{1}$$

where $x(t)$, x_0, and $\mathbf{A}$ denote the $n \times 1$ state vector, the $n \times 1$ initial states, and the $n \times n$ system matrix, respectively. The solution of (1) is

$$x(t) = e^{\mathbf{A}t}x_0 \tag{2}$$

The state vector $x(t)$ is obtained as

$$x(t) = \sum_{i=1}^{n} \alpha_i \, e^{\lambda_i t} \, \mathbf{v}_i \tag{3}$$

Eq. (3) is referred to as the Modal Expansion Theory (Kailath, 1980; Ogata, 2003). From (3), the unforced system response $x(t)$ depends upon λ_i (the ith eigenvalue), $\mathbf{v}_i$ (the corresponding ith eigenvector) and α_i (a constant associated with initial conditions). Each term of $\exp(\lambda_i t)\mathbf{v}_i$ is referred to as a *mode* and $x(t)$ is a composite response formed from the linear combination of every mode $\exp(\lambda_i t)\mathbf{v}_i$ with the initial state related scalar term α_i as the coefficients. A real eigenvalue corresponds to a non-oscillatory mode. A positive real eigenvalue represents an aperiodic unstable mode, and a negative real eigenvalue represents a decaying mode. On the other hand, complex eigenvalues occur in conjugate pairs and each pair corresponds to an oscillatory mode. A pair of complex eigenvalues $\lambda = \sigma \pm j\omega$ includes a real part σ and an imaginary part ω. The imaginary part $\omega = 2\pi f$ gives the frequency f of the corresponding oscillatory mode. The real part σ reveals the damping of the associated oscillatory mode: a positive value means a negative damping while a negative value represents a positive damping. A real part with zero value implies there is no damping with the mode. A linear system is stable if every eigenvalue of its system matrix has a negative real part.

IV. THE CALCULATION METHOD FOR SMALL SIGNAL STABILITY ANALYSIS

In order to calculate the small signal stability of the power system, we select a shift value based on the combination of S-matrix method, shift method, order reduction method and power method in order to solve the most critical eigenvalue of the system state matrix of the large power system, and various methods are described below (Campagnolo et al., 1995; Lima et al., 1995; Makarov et al, 1998; Uchida and Nagao, 1988).

1. Power Method

The basic idea that underlies almost every partial eigenvalue computation method is that the sequence $\mathbf{x}, \mathbf{Ax}, \ldots, \mathbf{A}^k\mathbf{x}$ converges to the eigenvector $\mathbf{v}_1$ associated with the eigenvalue of largest modulus (λ_1) of matrix $\mathbf{A}$, provided that $|\lambda_1| > |\lambda_2| \geq \ldots \geq |\lambda_n|$. The convergence of this method is linear and depends on the ratio $|\lambda_1|/|\lambda_2|$.

This method is not suitable for a direct application to the small signal stability analysis, since the modes of interest in this problem are not those with largest moduli in the state matrix $\mathbf{A}$. The inverse iteration method has been successfully applied to the small-signal stability analysis. This method uses the matrix transform.

$$\mathbf{M}_1 = f_1(\mathbf{A}) = (\mathbf{A} - q\mathbf{I})^{-1} \tag{4}$$

where q is a complex shift, in place of the matrix $\mathbf{A}$, in the power sequence. The eigenvalues of $\mathbf{A}$ closest to q will be mapped to the eigenvalues of largest moduli in $\mathbf{M}_1$ and thus the convergence will be driven to these eigenvalues and respective eigenvectors.

2. S-Matrix Method

The S-matrix method (Uchida and Nagao, 1988) may be generalized into the matrix transform

$$\mathbf{M}_2 = f_2(\mathbf{A}) = \left(\mathbf{A} + \bar{h}\,\mathbf{I}\right)\left(\mathbf{A} - h\,\mathbf{I}\right)^{-1} \tag{5}$$

where h is a complex number and $\bar{h}$ is its complex conjugate.

Despite its initial application with the Lanczos method, this matrix transform could also be used with the power method to converge to the eigenvalue of largest modulus in $\mathbf{M}_2$. The maximum eigenvalue (maximum absolute value) of $\mathbf{M}_2$ and the corresponding eigenvector can be solved by using the power method. The relationship between the eigenvalue after S-matrix transformation (λ') and the eigenvalue of the original matrix (λ) is

$$\lambda' = \frac{\lambda + \bar{h}}{\lambda - h} \tag{6}$$

3. The Calculation of the Most Critical Eigenvalue of the Small Signal Stability

The aforementioned method is not capable of directly solving the most critical eigenvalue of system matrix $\mathbf{A}$. Therefore, in this paper we introduce a method for swift determination of system stability which can solve the most critical eigenvalue (or with the lowest damping) of the system and the corresponding eigenvector. This method can also swiftly solve all unstable eigenvalues and corresponding eigenvectors without consuming too much time.

With the unstable eigenvalues of the system only accounting for a small portion of all eigenvalues, in this paper we have adopted a new calculation method for critical eigenvalue and eigenvector in order to determine and analyze the small signal stability of the electric power system. This calculation method is based on the “Power Method”, “S-matrix method” and “Shifting” method for solving the eigenvalues with the lowest damping and all unstable eigenvalues of the matrix of electric power system (Lima et al., 1995; Makarov et al., 1998; Gomes et al., 2003). When the eigenvalue of the electric power system is less than the threshold value, the output will be the eigenvalue with the lowest damping. When the eigenvalue of the electric power system is more than the threshold value, the outputs will be all unstable eigenvalues, alleviating the inconvenience for calculating all eigenvalues. In this paper we will also verify the feasibility of this algorithm by empirical analysis.

In an actual electric power system, the system matrix can be so enormous that the matrix processing is quite time consuming. Therefore, there have been literatures discussing all possible methods for solving the partial critical eigenvalues near certain shifting points (Lima et al., 1995; Makarov et al., 1998; Gomes et al., 2003), while there have also been continuous improvement on these methods. In this paper we have greatly improved the method for solving unstable eigenvalues. We have successfully solved the most unstable (or with the lowest damping) eigenvalue and the corresponding eigenvector based on an actual feasible method, which in turn can help us understand the behavior of system oscillation.

4. Shift Transform Method

The shift transform method is a combination of the shift method and the S-matrix method while the relationship is shown as follows:

$$\mathbf{M}_3 = f_3(\mathbf{A}) = [f(\mathbf{A}-\beta\mathbf{I})+\bar{h}\mathbf{I}][f(\mathbf{A}-\beta\mathbf{I})-h\mathbf{I}]^{-1} \quad (7)$$

Matrix **A** will become $\mathbf{M}_3$ after the transformation, and the critical eigenvalue (with the maximum absolute value) and the corresponding eigenvector of $\mathbf{M}_3$ can be solved by the power method. The relationship between the eigenvalue after matrix transformation (λ') and the eigenvalue of the original matrix is show below (λ):

$$\lambda' = \frac{\lambda + (\bar{h}-\beta)}{\lambda - (h+\beta)} \quad (8)$$

5. The Calculation of the Most Critical Eigenvalue

The calculation method for the most critical eigenvalue in this paper is described below:

Step 1. Apply shift transformation to the dynamic matrix **A** of the electric power system, such as

$$f_1(\mathbf{A}) = \mathbf{A} - \beta\mathbf{I} \quad (9)$$

Step 2. Apply S-matrix transformation to the shifted matrix $f_1(\mathbf{A})$, such as

$$f_2(\mathbf{A}) = [f_1(\mathbf{A}) + \bar{h}\,\mathbf{I}][f_1(\mathbf{A}) - h\,\mathbf{I}]^{-1} \quad (10)$$

Step 3. Solve the critical eigenvalue by using the power method.
Step 4. Zeroing of the critical eigenvalue.
Step 5. The matrix is shifted again to the location of that critical eigenvalue.
Step 6. Solve the current critical eigenvalue.
Step 7. Compare the real part of the current critical eigenvalue with the real part of previous critical eigenvalue. If the real part of current critical eigenvalue is greater than the real part of previous critical eigenvalue, step 6 will be repeated to solve the critical eigenvalue again.

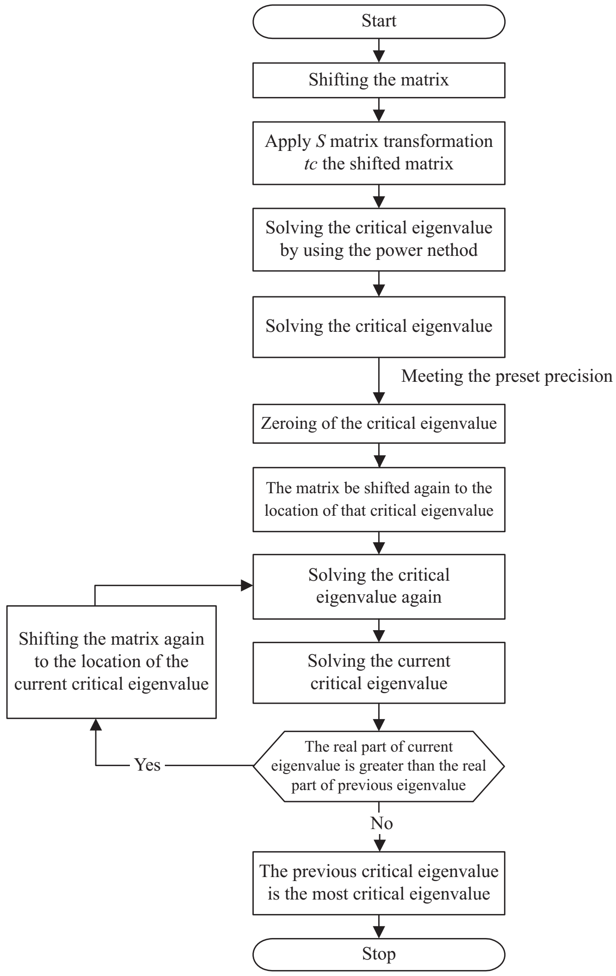

Fig. 1. The flow chart of solving the most critical eigenvalue.

Step 8. The previous critical eigenvalue will be the most critical eigenvalue.

Fig. 1 shows the flow chart of the method solving the most critical eigenvalue.

V. STUDY SYSTEM

In this paper we investigate a shipboard electric power system composed of two diesel power generators (represented by G_1 and G_2), electric power distribution network, and all ship electric power loads as the study system (National Taiwan Ocean University, 1993; Fang, 1997; Fang et al., 1998; Fang et al., 1999). With variety of operation conditions of loads on a ship, in this paper we have selected four frequently seen operation conditions (such as ship docking and undocking, entering and leaving port, normal navigation, and research operation) as examples for a quick determination and analysis of small signal stability of the shipboard electric power system.

Operation Condition 1: Docking and Undocking

During ship undocking, bow thruster must be activated (the

Table 1. All oscillatory mode eigenvalues of operation condition 1.

Mode	Eigenvalue
1	-6.1256 ± j38.970
2	-5.8617 ± j39.912
3	-2.1299 ± j16.033
4	-1.5976 ± j1.4819*

* most critical eigenvalue

Table 2. All oscillatory mode eigenvalues of operation condition 2.

Mode	Eigenvalue
1	-6.4564 ± j38.971
2	-2.0629 ± j16.072*

* most critical eigenvalue.

Table 3. All oscillatory mode eigenvalues of operation condition 3.

Mode	Eigenvalue
1	-6.4656 ± j38.968
2	-2.0996 ± j16.014*

* most critical eigenvalue.

motor is represented by $\mathbf{M}_2$) with the single one diagram as shown in Fig. 2, where G_1 and G_2 represent the two power generators on the ship, $\mathbf{M}_1$ represents the motor equivalent to all dynamic loads under such operation condition, TR represents the 440/220V transformer, and Load represents the static load. The system state matrix **A** is a 23 × 23 matrix. After matrix **A** is substituted into the critical eigenvalue calculation program, the most critical eigenvalue can be obtained as (-1.5976 ± j1.4819), while all oscillatory mode eigenvalues of the system are listed in Table 1.

Operation Condition 2: Entering and Leaving Port

After undocking, the ship still needs to sail for a while within the port so that the two generators must be activated simultaneously in order to ensure system safety. One of the two generators can be turned off once the ship has left the port. The one line diagram isshown in Fig. 3. The system state matrix **A** is a 12 × 12 matrix. After matrix **A** is substituted into the critical eigenvalue calculation program, the most critical eigenvalue can be obtained as (-2.0629 ± j16.072), while all oscillatory mode eigenvalues of the system are listed in Table 2.

Operation Condition 3: Normal Navigation

The one line diagram of the shipboard electrical power system during normal navigation is shown in Fig. 4. The state matrix after the linearization of the system is a 12 × 12 matrix. After the system matrix is substituted into the critical eigenvalue calculation program, all unstable eigenvalues can be obtained as (-2.0996 ± j16.014), while all oscillatory mode eigenvalues of the system are listed in Table 3.

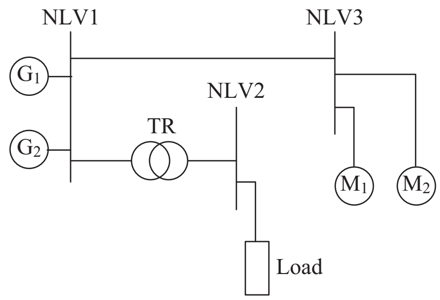

Fig. 2. The equivalent electric single wire diagram of entering and leaving port (operation condition 1).

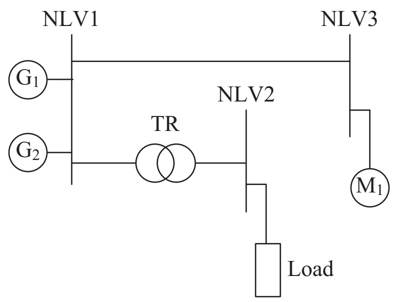

Fig. 3. The equivalent electric single wire diagram of entering and leaving port (operation condition 2).

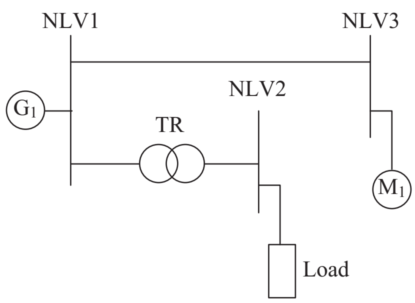

Fig. 4. The equivalent electric single wire diagram of normal navigation (operation condition 3).

Operation Condition 4: Research Operation

When the ship arrives at the local place for research, two marine research winches will be activated to conduct the research work. They will be turned off after the task has been completed. The one line diagram under such an operation is shown in Fig. 5. The state matrix after the linearization of the system is a 25 × 25 matrix. After the obtained matrix is substituted into the critical eigenvalue calculation program, all unstable eigenvalues can be obtained as (-1.4472 ± j1.1896), while all oscillatory mode eigenvalues of the system are listed in Table 4.

Comparison for All Operation Conditions

According the calculation results with respect to the four

Table 4. All oscillatory mode eigenvalues of operation condition 4.

Mode	Eigenvalue
1	$-29.156 \pm j2.1197$
2	$-6.4074 \pm j38.921$
3	$-5.9762 \pm j39.627$
4	$-1.9135 \pm j16.352$
5	$-1.4472 \pm j1.1896^{*}$

* most critical eigenvalue.

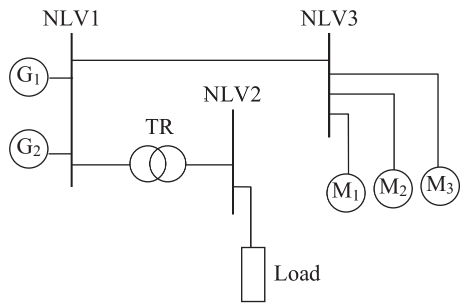

Fig. 5. The equivalent electric single wire diagram of research operation (operation condition 4).

operation conditions, we find that the most critical eigenvalues during the normal navigation is closer to the right half-plane of the complex plane than all other operation conditions. This is because that one power generator is shut down during normal navigation as the backup power source. The motors on the ship have accounted for greater system power supply capacity such that the relatively smaller system capacity due to only one power generator as the power supply will definitely lead to a reduced margin of system stability. As for the two operation conditions of "docking and undocking" and "entering and leaving port", even though the load variation is more drastic and two or one propulsion motor must be activated, they can still be more stable due to simultaneous activation of two power generators.

In summary, out of the four operation conditions, we can infer that the system is relatively close to the stable limit during normal navigation from the perspective of small signal stability. For preventing system instability due to greater load or other factors or even the system collapse, special attention must be paid during normal navigation to eliminate all possible causes of disturbances and accidents, or relevant equipments are to be installed in order to improve system stability.

VI. CONCLUSION

The main purpose of this paper is to propose an effective method for the calculation of the most critical eigenvalue (or the eigenvalue with the lowest damping) of the shipboard electric power system. The proposed algorithm focuses on calculating the critical eigenvalue to alleviate the time consumption for finding all eigenvalues such that the goal of rapid analysis of small signal stability of the shipboard electric power system can be achieved. A sample shipboard electric power system has been utilized as an example to calculate and investigate the system critical eigenvalues under four typical operating conditions.

By using the method, we don't have to guess and try the shift point to estimate the critical eigenvalue. We can solve and confirm the eigenvalue (with the lowest damping) of the shipboard electric system matrix, and based on this eigenvalue we can determine whether the system is stable or not. The method can be used to solve the most critical eigenvalue of the system and the corresponding eigenvector. This is an improvement over the traditional method which can only be used for solving critical eigenvalues rather than solving the most critical eigenvalue.

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