Journal of Marine Science and Technology
DOI: 10.6119/JMST-012-0206-8

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ON THE STABILIZATION OF A CLASS OF UNCERTAIN SYSTEMS WITH TIME-VARYING DELAY VIA VSC APPROACH

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Key words: Time-delayed systems, variable structure control, time-varying unmatched uncertainties.

ABSTRACT

In this paper, the robust stabilization problem for a class of time-delayed multi-input systems with matched and unmatched uncertainties is considered, here, all the matched uncertainties, unmatched uncertainties and the delay term may be time-varying. Variable structure control (VSC) is suggested to design stabilizing controllers for these uncertain time-delayed multi-input systems. The proposed controllers guarantee the global reaching condition of the sliding mode in these uncertain time-delayed multi-input systems. Further, they ensure that the system trajectories asymptotically converge to the sliding mode. In the sliding mode, the investigated time-delayed systems with matched and unmatched uncertainties still possess the insensitivity to the uncertainties and external disturbances, which is the same as the linear systems do. And the proposed controllers can work effectively for systems either with matched or unmatched uncertainties. However, the above desirable properties cannot be guaranteed by the traditional VSC design for the systems without time delays or with matched uncertainties. Finally, an illustrative example is included to demonstrate the effectiveness of the proposed variable structure controller.

I. INTRODUCTION

Time delay is frequently found in various physical systems, for example, in long transmission lines, in hydraulic or pneumatic systems, electrical networks, chemical process, and so on. The stabilization problem of uncertain control systems with time delay in the state has become an important issue in recent years. It is in anticipation of the poor system performance or even instability may arise from the existence of time delay, and the existence of time delay renders the control problem much more complex and difficult. Using the different methods, the problem of the stabilization for uncertain time-delay systems has been studied in [1-9]. For example, the problem of the global delay-dependent robust stability in the mean square for uncertain stochastic neural networks with time-varying delay is investigated by Lu et.al. [1]. A a global robust stability analysis of a particular class of hybrid bidirectional associative memory time-varying delayed neural network with norm-bounded time varying parameter uncertainties is studied in [2]. In [3], the problem of adaptive robust control for uncertain linear systems with delay occurring in the state variables is studied. Hua et.al. [4] studied a state feedback control problem for a class of nonlinear time-delayed via the backstepping method. In [5], Shyu, Liu and Hsu developed a new robust control law to stabilize an uncertain large-scale time delayed system with a dead-zone input. Hsu [6] discussed the time delayed problem for some uncertain systems and large-scale systems with nonlinear input. In [7], some new separated $H\infty$ and H_2 performance criteria are derived for a class of time-delay systems. Liu [8] studied the observer design for nonlinear uncertain time-varying delay systems with unmatched uncertainties. And the equivalence and efficiency of certain stability criteria for time-delay systems are discussed in [9]. In the above literatures, for the uncertainty part, the matching condition is required for some of them, for the time delay part, some of them needed time delay is constant or the derivative of the time-varying delay is known or even the derivative of the delay has to be less than one. However, in practice, the uncertainties of the systems may be unmatched and the delay of systems may be time varying and the derivative of the delay may not be less than one.

In this paper, VSC is suggested to design stabilizing controllers for the time-delayed multi-input systems with matched and unmatched uncertainties, here, all the matched uncertainties, unmatched uncertainties and the delay term may be time-varying. For robust control system, VSC is habitually adopted due to its congenital advantages [10]-[11], that is, order reduction and robustness with respect to matched parameters and/or external disturbances. The famous invariant property of VSC says that if the uncertainties of the system satisfy the matching condition, then the system behavior in the

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sliding mode is insensitive to the matched uncertainties and the system is governed by the predetermined switching surface. However, if the matching condition is not satisfied, that is, the unmatched uncertainties exist in the developed system, then the system behavior in the sliding mode can not be governed by the predetermined switching surface. Motivated by the existence of time delay and unmatched uncertainties, in this paper, a new VSC law is proposed to deal with the time-delayed system with matched and unmatched uncertainties.

The rest of the paper is organized as follows. System and assumption statements are introduced in Section II. The system dynamics analysis on the sliding surface is presented in Section III. Section IV elaborates the new variable structure control law. Illustrate example and concluding remarks are given in Sections V and VI, respectively. Without loss of generality, throughout the remainder of this paper, $(*)^T$ denotes the transpose of (*), $\|(*)\|$ denotes the Euclidean norm when (*) is a vector, or the Frobenius norm when (*) is a matrix..

II. SYSTEM DESCRIPTION

Consider a class of multi-input systems with time-varying matched uncertainty, unmatched uncertainty and time-varying delay, which are described by the following equations

$$\dot{x}(t) = (A+\Delta A(t))x(t) + A_d x(t-d(t)) + Bu(t) + \Delta f(t).$$
$$x(t) = \theta(t), \quad \text{for } -d(t) \le t < 0. \tag{1}$$

where $x(t) \in R^n$ and $u(t) \in R^m$ are the vectors of the state variables and control inputs of the uncertain time-delayed system, respectively. $\Delta f(t) \in R^n$ is a vector of the external nonlinear disturbances of the system, $d(t)$ stands for a positive nonzero time-varying delay. $\theta(t)$ represents a continuous vector-valued initial function. $A \in R^{n\times n}$, $A_d \in R^{n\times n}$ and $B \in R^{n\times m}$ are the nominal state matrix, state-delay term matrix and the input matrix of the uncertain system, respectively. The matrix $\Delta A(t)$ is the unknown time-varying system parameter uncertainty which is considered to be unmatched in this paper. Before deriving our proposed approach, the following assumptions are introduced for system (1).

Assumption 1. The matrix pair (A, B) is controllable, i.e., there exists a matrix K, such that $\underline{A} = A - BK$ is stable.

Assumption 2. There exists a vector $f(t) \in R^m$ and known non-negative constant β_f, such that

$$\Delta f(t) = Bf(t) \text{ with } \|f(t)\| \le \beta_f. \tag{2}$$

Assumption 3. The uncertain time-varying state matrix $\Delta A(t)$ is not matched, i.e., it is the unmatched uncertainty with the form

$$\Delta A(t) = EF(t)G. \tag{3}$$

Here, E and G are known constant matrices with appropriate dimension and $F(t)$ is unknown uncertain matrix, but satisfies $F^T(t)F(t) \le I$ and I is an identity matrix with appropriate dimension.

III. SYSTEM'S DYNAMICS ON THE SLIDING SURFACE

According to the two VSC standard design steps, in this section, a proper switching surface will be chosen as a function of the system states, such that the equivalent system during sliding mode exhibits desired dynamic behavior. And in the next section, a new VSC law will be designed to force the system state trajectory to the sliding surface and hold it here for all subsequent time.

The associated switching surfaces of system (1) are specified as follows

$$S(t) = Cx(t). \tag{4}$$

where $S = [s_1\ s_2\ \ldots\ s_m]^T$ is a vector of switching surfaces, and $C = [c_1\ c_2\ \ldots\ c_m]^T$ is a specified constant m by n matrix.

Note that the system dynamics on the sliding surfaces have the desired responses if C is selected appropriately, and the system dynamics satisfy the equations given below

$$S(t) = 0 \text{ and } \dot{S}(t) = 0. \tag{5}$$

Substituting (1) into (5), one gets

$$\dot{S}(t) = C[(A+\Delta A(t))x(t) + A_d x_d + Bu(t) + \Delta f(t)] = 0. \tag{6}$$

where x_d denotes $x(t-d(t))$.

The matrix product CB is chosen to be nonsingular to derive the equivalent control $u_{eq}(t)$, that is

$$u_{eq}(t) = -(CB)^{-1}C[(A+\Delta A(t))x(t) + A_d x_d + \Delta f(t)]. \tag{7}$$

Hence, using (2), (3) and (7), the developed system (1) restricted to the sliding surfaces are in the form of

$$\dot{x} = [I - B(CB)^{-1}C]\,[(A+EF(t)G)\,x + A_d x_d]. \tag{8}$$

Remark 1. From the last equation, one can find the fact that the system dynamics can not be dominated by the sliding mode, and the traditional VSC just required the uncertainties satisfy the matching condition will limit the application of the VSC approach. In other words, the well-known invariance condition of VSC is not held for the time-delayed systems with unmatched time-varying uncertainties.

In order to overcome the fact stated in *Remark* 1, fortunately, the so-called sliding coefficient matching (SCM) condition proposed in [12] could be used to make the invariant property also hold for the time-delayed systems with unmatched time-varying uncertainties, and the following Definition is made for the SCM condition.

Definition. The uncertainty $\Delta A(t) = EF(t)G$ is said to satisfy the SCM condition, if there exists a matrix H with appropriate dimension, and the matrix G satisfies

$$G = HC. \quad (9)$$

Remark 2. The poor system performance or even instability may arise from the presence of uncertainties and/or disturbances. Fortunately, they may be overcome if they satisfy the traditional matching (TM) condition, that is, the uncertainties and/or disturbances are within the range space of the input matrix B. Unfortunately, the system cannot be forced to the sliding mode if the uncertainties and/or disturbances are NOT within the range space of the input matrix B, that is, they don't satisfy the TM condition (see Figure 2). However, the SCM condition says that we can deal with such unmatched uncertainties by selecting matrix C appropriately. Although SCM condition looks like TM condition, the SCM condition is more flexible than the TM condition.

Using (2), one can easily derive the system restricted to the sliding surfaces as shown in (8)

$$\dot{x} = [I - B(CB)^{-1}C]\,[(A + EF(t)G)\,x + A_d x_d]. \quad (10)$$

The SCM condition shown in (9) could be used to conquer the unmatched time-varying matrix $\Delta A(t)$. Then,

$$\Delta A(t) = EF(t)G = EF(t)HC. \quad (11)$$

Hence, when the developed system is on the sliding surface, one gets

$$\dot{x} = [I - B(CB)^{-1}C]\,(A\,x + A_d x_d). \quad (12)$$

From the above equation, one can see that the system dynamics are not affected by the unmatched part. Nevertheless, the stabilization problem of such a time-delayed system has been solved by a good existing paper, which is proposed by Richard [13]. Thus, in the sliding mode, the investigated time-delayed systems still possess the insensitivity to the matched external disturbances and/or unmatched time-varying system parameter uncertainties. Furthermore, a new VSC law will be designed in next section, so that the system dynamics (12) will be driven to the sliding surface and stayed thereafter.

IV. VARIABLE STRUCTURE CONTROL LAW DESIGN

According to the second VSC standard design phase, a new VSC law will be designed in this section, so that the system states are forced to the sliding surface and stayed here for all subsequent time. The following lemmas are needed to achieve a new VSC law to drive the unmatched uncertain time-delayed system trajectories into the sliding mode.

Lemma 1. If the following condition is held, then the motion of the sliding mode (4) is asymptotically stable

$$S^T(t)\,\dot{S}(t) < 0, \quad \forall\ t \geq 0. \quad (13)$$

proof . Let the Lyapunov function candidate of the system (1) be chosen as

$$V(t) = \frac{1}{2} S^{\mathrm{T}}(t) S(t). \quad (14)$$

Then condition (13) ensures that

$$\dot{V}(t) = S^{\mathrm{T}}(t)\,\dot{S}(t) < 0. \quad (15)$$

Hence, the system trajectories are toward the switching surfaces and the sliding mode (5) is asymptotically stable. *

Lemma 2 [12]. If $F^{\mathrm{T}}(t)\,F(t) \leq I$, then

$$2a^{\mathrm{T}}F(t)b \leq a^{\mathrm{T}}a + b^{\mathrm{T}}b, \ \forall\ a, b \in R^n. \quad (16)$$

In order to meet the condition (13), the new appropriate VSC law is proposed as follows

$$u(t) = \frac{-B^{\mathrm{T}}C^{\mathrm{T}}S}{\left\|B^{\mathrm{T}}C^{\mathrm{T}}S\right\|} R(x,t) - \frac{1}{2}(CB)^{-1}[CEE^{\mathrm{T}}C^{\mathrm{T}}S + H^{\mathrm{T}}HS], \quad (17)$$

where

$$R(x,t) = \eta\{(\left\|(CB)^{-1}CA\right\| + r\left\|(CB)^{-1}CA_d\right\|)\|x\| + \beta_f\}, \ \text{with } \eta > 1 \text{ and } r > 1. \quad (18)$$

Now, we are in a position to establish the *Theorem* of this paper, which shows that the developed time-delayed systems with unmatched time-varying uncertainties can be forced on the sliding surface (4) by applying the proposed VSC laws (17).

Theorem 1. For the multi-input uncertain time-delayed system (1) with matched and unmatched uncertainties satisfying *Assumptions* 1 ~ 3. The system trajectories will asymptotically converge to the sliding surface (4), if the proposed VSC laws (17) applied to the developed system (1).

Proof.

Let the Lyapunov function of the system (1) be chosen as (14).

Substituting (1) and (4) into (14), and taking the time derivative of V, one gets

$$\dot{V}(t) = S^{\mathrm{T}}(t)\,\dot{S}(t)$$
$$= S^{\mathrm{T}}(t)C[(A + \Delta A(t))x(t) + A_d x_d + Bu(t) + \Delta f(t)]. \quad (19)$$

Using (2), (3), (9), (16) and the property $\|PQ\| \leq \|P\|\|Q\|$, one has

$$\begin{aligned}\dot{V}(t) &= S^{\mathrm{T}}(t)C[Ax(t) + A_d x_d + Bf(t)] \\ &\quad + S^{\mathrm{T}}(t)CEF(t)Gx(t) + S^{\mathrm{T}}(t)CBu(t) \\ &\leq \left\|S^{\mathrm{T}}CB\right\| [\left\|(CB)^{-1}CA\right\|\|x\| + \left\|(CB)^{-1}CA_d\right\|\|x_d\| + \beta_f] \\ &\quad + \frac{1}{2}[S^{\mathrm{T}}CEE^{\mathrm{T}}CS + S^{\mathrm{T}}H^{\mathrm{T}}HS] + S^{\mathrm{T}}CBu(t). \end{aligned} \quad (20)$$

Then, applying the proposed VSC laws (17), (18) and with the help of Razumikhin theorem [14], $\|x_d\| < r\|x\|$ for $r > 1$, one obtains

$$\dot{V}(t) < \|S^{\mathrm{T}}CB\| \, [\|(CB)^{-1}CA\|\|x\| + r\|(CB)^{-1}CA_d\|\|x\| + \beta_f]$$

$$-\eta\|S^{\mathrm{T}}CB\| \, [\|(CB)^{-1}CA\|\|x\| + r\|(CB)^{-1}CA_d\|\|x\| + \beta_f]$$

$$<(1-\eta)\,\|S^{\mathrm{T}}CB\|$$

$$[\|(CB)^{-1}CA\|\|x\| + r\|(CB)^{-1}CA_d\|\|x\| + \beta_f] \qquad (21)$$

Because $\eta>1$, it is clear to derive

$$\dot{V}(t) = S^{\mathrm{T}}(t)\,\dot{S}(t) < 0. \qquad (22)$$

From (22), based on the Lyapunov stability theory, one can see that the system dynamics are asymptotically converged to the sliding surface (4). *

V. ILLUSTRATIVE EXAMPLE [15]

In this section, an illustrative example is selected from the literature on the subject to demonstrate the effectiveness of the proposed variable structure controller. The following dynamical system is adopted from [15] except the delay part.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1+\zeta & 0\\ 0 & -1 & 1\\ 0 & -\zeta & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}1 & 1 & 0\\ -1 & -1 & 0\\ 0 & 1 & 1\end{bmatrix}\begin{bmatrix}x_1(t-d(t))\\ x_2(t-d(t))\\ x_3(t-d(t))\end{bmatrix}$$

$$+\begin{bmatrix}1 & 0\\ -1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \zeta\end{bmatrix}. \qquad (23)$$

where ζ is a time-varying variable ranging in [–1 1], and the initial values are given as $x(0)=[1\ 0\ {-1}]^{\mathrm{T}}$.

Comparing (23) with system (1), we get the following parameters

$$A = \begin{bmatrix}0 & 1 & 0\\ 0 & -1 & 1\\ 0 & 0 & 0\end{bmatrix},\ \Delta A(t) = \begin{bmatrix}0 & \zeta & 0\\ 0 & 0 & 0\\ 0 & -\zeta & 0\end{bmatrix},\ A_d = \begin{bmatrix}1 & 1 & 0\\ -1 & -1 & 0\\ 0 & 1 & 1\end{bmatrix},$$

$$B = \begin{bmatrix}1 & 0\\ -1 & 0\\ 0 & 1\end{bmatrix},\ \text{and}\ \Delta f(t) = \begin{bmatrix}0\\ 0\\ \zeta\end{bmatrix}.$$

Further, based on (2) and (3), we can find the following data

$$f(t) = \begin{bmatrix}0\\ \zeta\end{bmatrix},\quad E = \begin{bmatrix}1\\ 0\\ -1\end{bmatrix},\quad F(t) = \zeta,\quad \text{and}\quad G = [0\ \ 1\ \ 0]$$

and it is clear to see that $\Delta A(t)$ is the unmatched part of the system.

Furthermore, from the above data and (18), we choose the following values for the computer simulation: $\zeta = \sin(t)$, $\beta_f = 1.2$, $\eta = 1.2$ and $r = 1.2$.

We know that the time delays of the system often change due to the variable environment, so we consider the case that $d(t) = 2|\sin(t)|$ for system (23).

The switching surfaces are selected as

$$S(t) = Cx(t) = \begin{bmatrix}1 & 0 & 2\\ 1 & 1 & 2\end{bmatrix}x(t).$$

For comparisons, we consider the following cases.

Case 1: Applying the conventional VSC law to the system (23) with zero unmatched uncertainties and no time delay term, i.e., $\zeta = 0$ and $A_d = 0$.

Case 2: Applying the conventional VSC law to the system (23) with unmatched uncertainties and with time delay term, i.e., $\zeta \neq 0$ and $A_d \neq 0$.

Case 3: Applying the modified proposed VSC law to the system (23) with zero unmatched uncertainties and no time delay term, i.e., $\zeta = 0$ and $A_d = 0$.

Case 4: Applying the modified proposed VSC law to the system (23) with unmatched uncertainties and with time delay term, i.e., $\zeta \neq 0$ and $A_d \neq 0$.

For *Case* 1, $\zeta = 0$ and $A_d = 0$, the system (23) can be rewritten as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & -1 & 1\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}1 & 0\\ -1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix} \qquad (24)$$

Following the conventional VSC design procedure for this case, we have the following conventional VSC law for (24)

$$u_{\mathrm{con}}(t) = -\eta\frac{B^{\mathrm{T}}C^{\mathrm{T}}S}{\|B^{\mathrm{T}}C^{\mathrm{T}}S\|}\{\|(CB)^{-1}CA\|\|x\|\},\ \eta > 1 \qquad (25)$$

In order to eliminate the control chattering, the boundary layer condition is commonly used, and the last law is modified as follow

$$u_{\mathrm{con}}(t) = -\eta\frac{B^{\mathrm{T}}C^{\mathrm{T}}S}{\|B^{\mathrm{T}}C^{\mathrm{T}}S\| + \varepsilon}\{\|(CB)^{-1}CA\|\|x\|\},\ \eta > 1 \qquad (26)$$

where $\varepsilon = 0.001$.

Applying the modified conventional law (26) to the system (24), one gets the well state trajectory performances shown in Figure 1. However, for *Case* 2, when the modified conventional law (26) is applied to system (23) with $\zeta \neq 0$ and $A_d \neq 0$, it does not ensure the stability of the time-delayed system with the unmatched uncertainties and the poor performances are shown in Figure 2.

Also, under the boundary layer condition, the proposed VSC law (17) is modified as follow to eliminate the control chattering.

$$u(t) = \frac{-B^{\mathrm{T}}C^{\mathrm{T}}S}{\left\|B^{\mathrm{T}}C^{\mathrm{T}}S\right\| + \varepsilon} R(x,t) - \frac{1}{2}(CB)^{-1}[CEE^{\mathrm{T}}C^{\mathrm{T}}S + H^{\mathrm{T}}HS] \tag{27}$$

where $\varepsilon = 0.001$.

Based on the SCM condition, the matrix H can be cast as $H = [-1\ 1]$. Now, for *Case* 3 and *Case* 4, the modified proposed VSC law (27) is applied to both (23) and (24) and the state trajectories are shown in Figure 3 and Figure 4, respectively. From these results, one can see that the proposed VSC law can work effectively for a class of time-delayed system no matter it is with or without unmatched uncertainties. However, the conventional VSC law is not applicable to the time-delayed system with the unmatched uncertainties. Furthermore, Figure 5 shows the time responses of the conventional VSC law (26) under the boundary layer condition and the time responses of the proposed VSC law (27) under the boundary layer condition are shown in Figure 6. From Figure 5 and Figure 6, one can clearly see that the control chattering phenomenon has been eliminated.

VI. CONCLUSIONS

In this paper, a new robust control law is proposed for a class of time-delayed multi-input systems with matched and unmatched uncertainties through variable structure control theory. We summarize this paper with the following results:
(a) The proposed VSC controllers can drive the trajectories of the investigated systems onto the sliding mode. (b) The investigated time-delayed multi-input system still bears the insensitivity to the unmatched uncertainties and disturbances, which is the same as the systems with matched uncertainties do. (c) For the time delay part, it needed no more time delay is constant and/or the derivative of the time-varying delay has to be less than one. (d) The proposed controllers can be successfully applied to uncertain time-delayed systems either with matched uncertainties or with unmatched uncertainties. However, this cannot be guaranteed by the conventional variable structure control, which is designed for the systems with matched uncertainties.

ACKNOWLEDGEMENTS

The author would like to thank Editor and the anonymous reviewers for their constructive comments to improve this paper. In addition, this work was supported by the National Science Council of Taiwan, under contract NSC-101-2221-E-252-003-.

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FIGURE CAPTIONS

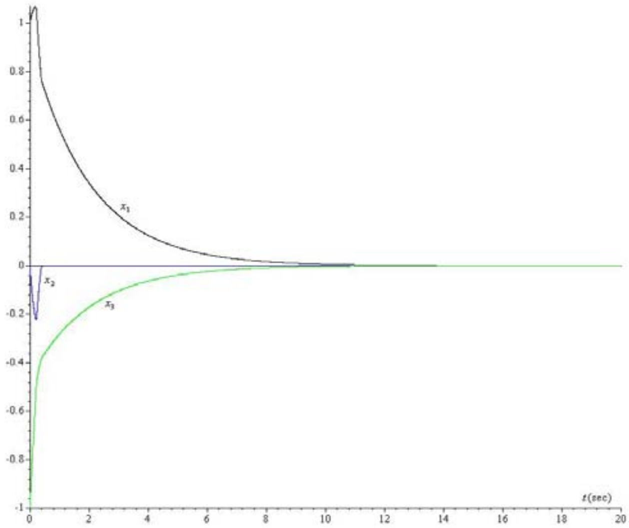

Figure 1. Applying (26) to the system (23) with $\zeta = 0$ and $A_d = 0$: States x_1, x_2, x_3 versus time.

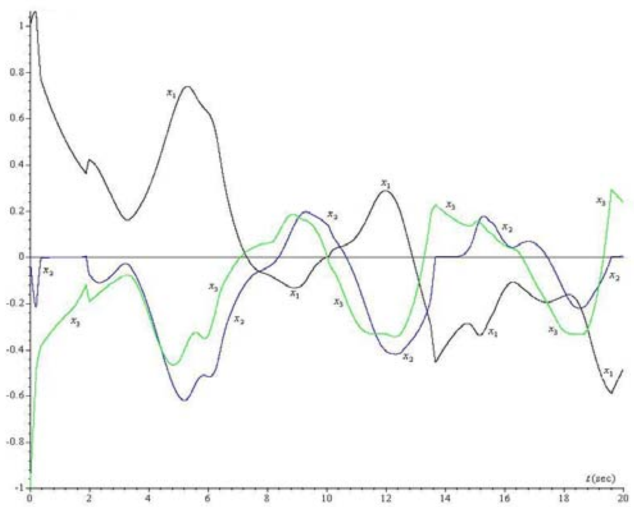

Figure 2. Applying (26) to the system (23) with $\zeta \neq 0$ and $A_d \neq 0$: States x_1, x_2, x_3 versus time.

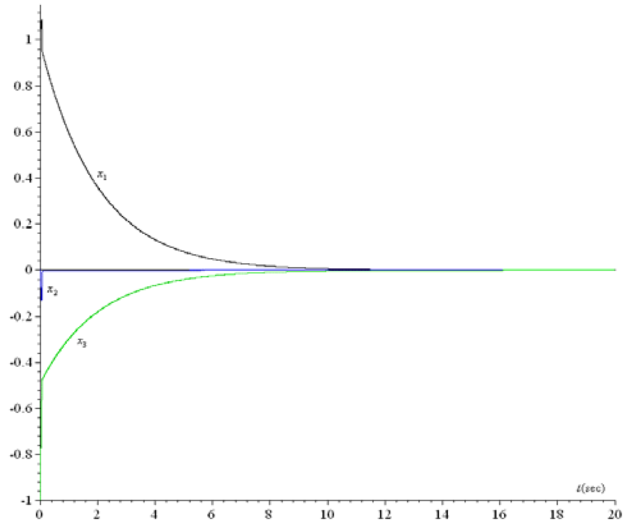

Figure 3. Applying (27) to the system (23) with $\zeta = 0$ and $A_d = 0$: States x_1, x_2, x_3 versus time.

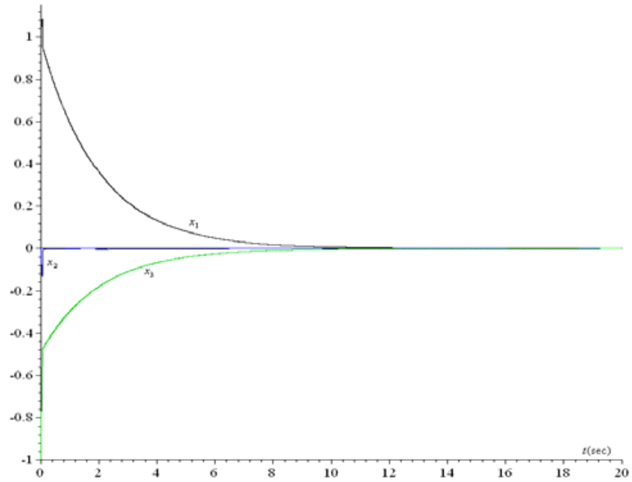

Figure 4. Applying (27) to the system (23) with $\zeta \neq 0$ and $A_d \neq 0$: States x_1, x_2, x_3 versus time.

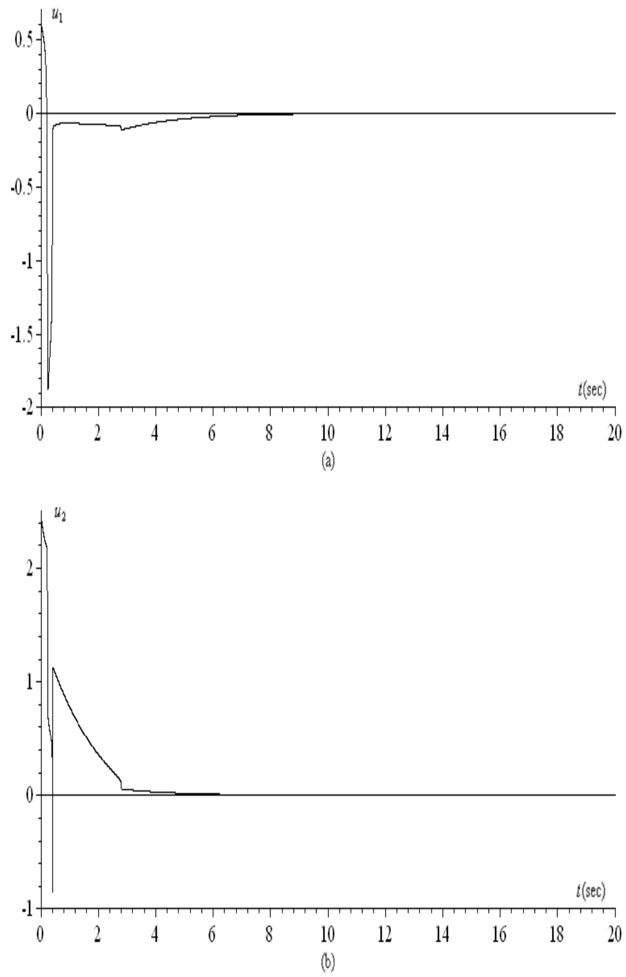

Figure 5. Time response of the conventional VSC law (26): (a) u_1 (b) u_2.

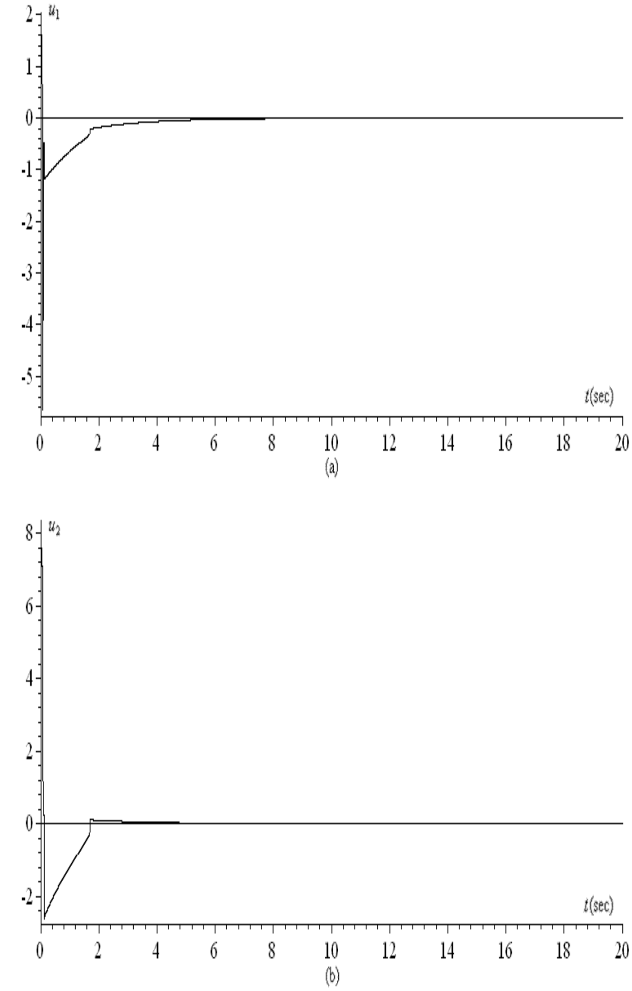

Figure 6. Time response of the proposed VSC law (27): (a) u_1 (b) u_2.