A VENDOR-BUYERS INTEGRATED INVENTORY MODEL INVOLVING QUALITY IMPROVEMENT INVESTMENT IN A SUPPLY CHAIN

Ming-Feng Yang[1] Ming-Cheng Lo[2] and Ta-Ping Lu[3]

ABSTRACT

Today's supply chain environment requires a new spirit of cooperation between the buyer and the vendor. So the characteristics of current manufacturing systems are consistent high quality, small lot sizes, frequent delivery, and close supplier ties. Just-in-time (JIT) production is an integrated set of activities to achieve those objects by adopting continuous quality improvement and developing a long-term partnership. This paper presents the optimum joint economic lot size in a case where multiple buyers are demanding one type of item from a single vendor by summing the ordering/setup cost, holding cost and quality improvement investment. This model is useful particularly for integrated inventory systems where the vendor and the buyer form a strategic alliance for profit sharing.

I. INTRODUCTION

In the current supply chain management (SCM) environment, companies are using JIT production to gain and maintain a competitive advantage. JIT requires a spirit of cooperation between the buyer and the vendor, and it has been shown that forming a partnership between the buyer and the vendor is helpful in achieving tangible benefits for both parties [5]. In this complex environment, successful companies have devoted considerable attention to reducing inventory cost and improving quality simultaneously.

Goyal [2] suggested a joint optimum economic lot size model with the objective of minimizing the total relevant costs for both the vendor and the buyer, in which a cooperative arrangement is enforced by some contractual agreement. Baner-

jee [1] proposed an optimum economic lot size model by assuming that the vendor produces to order for a buyer on a lot-for-lot basis under deterministic conditions. Goyal [2] generalized the model of Banerjee [1] by relaxing the assumption of the lot-for-lot policy of the vendor and showed that his joint optimum economic lot size model where the vendor's economic production quantity per cycle is an integer multiple of the buyer's purchase quantity provides a lower or equal joint total relevant cost when compared to Banerjee's model [1] Goyal and Gupta [4] reviewed the related literature on models which provide a coordinating mechanism between the buyer and the vendor. Lu [11] relaxed Goyal's [3] assumption of completing a batch before a shipment is started and explored a model that allowed shipments to take place any time during the production cycle with the delivery quantity to the buyer is known. Due to the frequent shipping policy proposed by the above model, the transportation cost should be taken into account in the relevant cost to investigate the relationship between number of shipments and inventory level. Shi and Su [18] suggested an integrated inventory model from the retailer's perspective only, and have thus ignored the fact that the manufacturer might have no incentive to accept returns. Ha and Kim [6] proposed a single-buyer single-vendor integrated model, under deterministic conditions, for a single product with a multiple shipments strategy including transportation cost. Hill and Omar [7] considered a supply chain in which a 'vendor' supplies a product to a 'buyer'. Lin and Yeh [12] illustrate that the best supply chain system benefit can be through coordinating the buyer's ordering lead time and the vendor's pricing policy also consider the risk costs and the effect of imperfect quality.

However, since the previous studies mainly focused on single-buyer situations, they are incomplete models of real supply chain environments. One supply chain may consist of not only one buyer or a sole buyer who demands an item from a deferent location. Siajadi et al [19] addressed a single-vendor multiple-buyer integrated inventory model to minimize joint total relevant cost for both vendor and buyer with a multiple shipment policy.

The challenge for business today is to produce quality products or services efficiently. Just-in-time production seeks to eliminate scrap and rework in order to achieve the reduction

of waste. Buyers, who want to establish a long-term supplier partnership, should select the supplier which can provide a high level of consistent quality and delivery reliability. Certainly, there are many ways to develop a shared benefit partnership, for instance, a foundation of integrated inventory model. In order to investigate the relation between quality imperfection and optimal order lot sizes, the imperfect production situation is considered, that is, the quality-related costs have been included. Porteus [16] and Rosenblatt and Lee [17] first presented this kind of relationship significantly. According to Porteus [16], there are many abundant quality improvement related litera-tures, for example: see Keller and Noori [9], Hong and Haya [8], Ouyang and Chang [14], Yang and Pan [21] and Ouyang et al [15]. Kevin Hsu and Yu [10] considered an inventory model that vendor can offer a one-time-only price discount to motivate buyers to order a special quantity for imperfective items.

This paper presents Siajadi et al [20] models in which the production process is imperfect. Building upon the work of Siajadi et al [20], we simultaneously optimize the or-der/production cycle time, shipment numbers and process quality with the objective of minimizing the total relevant cost. A procedure for finding the optimal solution is developed, and three numerical examples are given to illustrate the advantages of this model.

II. NOTATIONS AND ASSUMPTIONS

To establish the proposed model, the following notation is used:

D_j : demand rate per unit per unit time for buyer j;
P : production rate per unit per unit time;
A_j : ordering cost per order for buyer j;
S : setup cost per lot;
T : ordering or production cycle time;
m : number of buyers;
n_j : number of shipments for buyer j;
H_v : vendor's holding cost per unit per unit time;
Hb_j : buyer's holding cost per unit per unit time for buyer j;
AT_j : transportation cost per shipment for buyer j;
i : the fractional per unit time opportunity cost per dollar invested in stocks;
j : index of buyers.

The major assumptions made in the paper are listed below:
(1) There is a single vendor and multiple buyers for a single product.
(2) The product is manufactured with a finite production rate P, and this rate is greater than the sum of the demand rate $(P > \sum_{j=1}^{m} D_j)$..
(3) The production lot size is an amount that is equal to the sum of order size from all buyers.
(4) The implementation of shipping policy for the first shipment follows a sequence, the vendor starts to deliver first goods to the buyer who is defined to be the first buyer, and then the defined second buyer's goods will be shipped and the third one follows and so on.
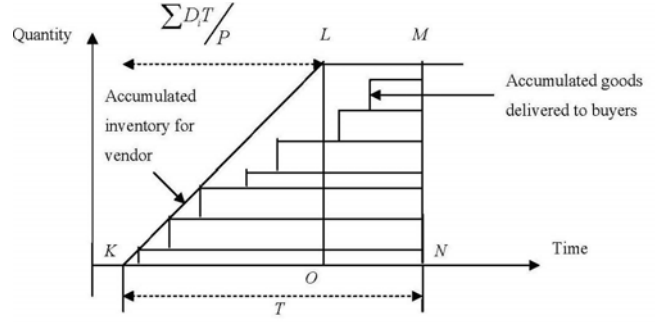

Fig. 1. The vendor's inventory pattern

(5) The production lot size is delivered in a number of equal-size shipments for each buyer, where the number and the size of the shipments for each buyer might be different.
(6) The order cycle time for each buyer and the production cycle time for each vendor is equal.
(7) Inventory is periodically reviewed and replenished.
(8) The replenishment lead time is of zero duration.
(9) No shortages are allowed.

The out-of-control probability θ is a decision variable, and is illustrated by a logarithmic investment function. The quality improvement and capital investment is illustrated by $q(\theta) = q \ln(\theta_0 / \theta)$ for $0 < \theta \le \theta_0$, where θ_0 is the current probability that the production process can go out of control, and $q = 1/\xi$, with ξ meaning the percentage decrease in θ per dollar increase in $q(\theta)$. The application of the logarithmic function on capital investment and quality improvement has been proposed by many authors, for example, Porteus [16], Hong and Hayya [8], Ouyang and Chang [14], Yang and Pan [21] and Ouyang et al [15].

III. BASIC MODEL

In this study, a model proposed by Siajadi et al [19] will be considered, which posits a situation where in a single vendor has a multiple-shipment policy to satisfy one buyer. Notice that the vendor's inventory pattern of this model is an extended pattern from Joglekar [13].

The vendor produces the item in the quantity of ΣDjT, and the buyer j will receive it in nj lots. The form of the vendor's inventory is shown in Figure 1. For the vendor, its average inventory can be evaluated as:

$$\overline{I_v} = \frac{T}{2P}\left[\left(\sum_{j=1}^{m} D_j\right)^2 + 2\sum_{j=1}^{m} D_j I_A - I_B\right] \tag{1}$$

Where

$$I_A = \frac{P}{n_1}\left(\frac{D_1}{P} + n_1 - 1\right) - \sum_{j=1}^{m} D_j$$

$$I_B = \sum_{j=1}^{m} D_j \left\{ \frac{2n_j\left[P(n_1-1) + D_1 - n_1\sum_{k=1}^{j}\frac{D_k}{n_k}\right] - Pn_1(n_j-1)}{n_1 n_j} \right\}$$

Therefore, the vendor's holding cost per unit time is

Holding cost $= H_V\left\{\dfrac{T}{2P}\left[\left(\sum_{j=1}^{m}D_j\right)^2+2\sum_{j=1}^{m}D_jI_A-I_B\right]\right\}$.

Since the vendor's initial setup cost is S, and the production cycle time is given by T. The total unitary time cost for the vendor is given by:

$$VTC=\frac{S}{T}+\left\{\frac{T}{2}\frac{H_V}{P}\left[\left(\sum_{j=1}^{m}D_j\right)^2+2\sum_{j=1}^{m}D_jI_A-I_B\right]\right\} \qquad (2)$$

On the buyers' side, the average inventory on hand during a cycle of buyer j is

$$\overline{I}_j=\frac{TD_j}{2n_j} \qquad (3)$$

Therefore, the buyers' total holding cost is

Holding cost $=\dfrac{T}{2}\sum_{j=1}^{m}H_{bj}\dfrac{D_j}{n_j}$

And the buyers' total ordering and transportation cost can be summed as:

Total ordering and transportation cost $=\dfrac{1}{T}\left[\sum_{j=1}^{m}\left(A_j+n_jA_{Tj}\right)\right]$

Hence, the total unitary time cost for the buyers is given by:

$$BTC=\frac{1}{T}\left[\sum_{j=1}^{m}\left(A_j+n_jA_{Tj}\right)\right]+\frac{T}{2}\sum_{j=1}^{m}H_{bj}\frac{D_j}{n_j} \qquad (4)$$

As the cost functions for the vendor and the buyers show above, the joint total cost is the sum of VTC and BTC as:

$$JTC=\frac{1}{T}\left(S+\sum_{j=1}^{m}\left(A_j+n_jA_{Tj}\right)\right)$$
$$+\frac{T}{2}\left\{\frac{H_v}{P}\left[\left(\sum_{j=1}^{m}D_j\right)^2+2\sum_{j=1}^{m}D_jI_A-I_B\right]+\left(\sum_{j=1}^{m}H_{bj}\frac{D_j}{n_j}\right)\right\}$$
$$=\frac{1}{T}\left[S+\sum_{j=1}^{m}\left(A_j+n_jA_{Tj}\right)\right]+\frac{T}{2}\left\{\frac{H_v}{P}\sum_{j=1}^{m}D_j\left(P-\sum_{j=1}^{m}D_j\right)\right.$$
$$\left.+\sum_{j=1}^{m}\left[\frac{D_j}{n_j}\left(\frac{2H_v}{P}\sum_{k=j}^{m}D_k+H_{bj}-H_v\right)\right]\right\} \qquad (5)$$

For the sake of including the situation of an imperfect production process, the assumption made in the model proposed by Porteus [16] is considered. The vendor produces the items by a quantity of ΣD_jT, the production process becomes out of control and begins to produce defective products, with a given probability of θ. Porteus [16] suggested the expected number of defective items in a run of size ΣD_jT can be evaluated as $T^2(\Sigma D_j)^2\theta/2$. Suppose there is an additional cost g to deal with defective units, then the expected defective cost per unit time is given by $gT(\Sigma D^j)^2\theta/2$.

Thus, the joint total unitary time cost incorporating the defective cost per unit time can be represented by

$$TC=JTC+\frac{gT\left(\sum_{j=1}^{m}D_j\right)^2\theta}{2} \qquad (6)$$

IV. IMPLEMENTATION INTEGRATED INVENTORY

MODEL INVOLVING QUALITY IMPROVEMENT INVESTMENT

In this paper, the effect of investment on quality improvement will be discussed as an integrated model whose objective is to minimize the sum of ordering/setup cost, holding cost, quality improvement and transportation cost by simultaneously determining the optimal values of T, nj and θ. Hence, according to Siajadi et al [20] the total relevant cost per unit time is

$$TRC=TC+iq\ln\frac{\theta_0}{\theta}$$
$$=\frac{1}{T}\left[S+\sum_{j=1}^{m}\left(A_j+n_jA_{Tj}\right)\right]$$
$$+\frac{T}{2}\left\{\frac{H_v}{P}\sum_{j=1}^{m}D_j\left(P-\sum_{j=1}^{m}D_j\right)+g\theta\left(\sum_{j=1}^{m}D_j\right)^2\right.$$
$$\left.+\sum_{j=1}^{m}\left[\frac{D_j}{n_j}\left(\frac{2H_v}{P}\sum_{k=j}^{m}D_k+H_{bj}-H_v\right)\right]\right\} \qquad (7)$$
$$+iq\ln\frac{\theta_0}{\theta}$$

Subject to
$$n_1,n_2,n_3,....,n_m\geq1,$$
$$\frac{T}{n_j}\geq\frac{T}{P}\sum_{j=1}^{m}\frac{D_j}{n_j}$$
and $0<\theta\leq\theta_0$.

There are three constraints which restrict equation (7), the first constraint is the requirement that each buyer must be shipped at least one time, the second one is the assurance of the assumption made above, that is, each buyers' first shipment has to be done in sequence, and the last constraint says that the optimal θ is an positive number and cannot be greater than θ0. In order to find the minimum cost for this problem, relax the second and the third constraints temporarily and take the first partial derivatives of TRC with T, nj and θ respectively as follows to minimize the total relevant cost function:

$$\frac{\partial TRC}{\partial T}=-\frac{1}{T^2}\left[S+\sum_{j=1}^{m}\left(A_j+n_jA_{Tj}\right)\right]+\frac{1}{2}\left\{\frac{H_v}{P}\sum_{j=1}^{m}D_j\left(P-\sum_{j=1}^{m}D_j\right)\right.$$
$$\left.+g\theta\left(\sum_{j=1}^{m}D_j\right)^2+\sum_{j=1}^{m}\left[\frac{D_j}{n_j}\left(\frac{2H_v}{P}\sum_{k=j}^{m}D_k+H_{bj}-H_v\right)\right]\right\} \qquad (8)$$

$$\frac{\partial TRC}{\partial\theta}=\frac{Tg\left(\sum_{j=1}^{m}D_j\right)^2}{2}-\frac{iq}{\theta} \qquad (9)$$

$$\frac{\partial TRC}{\partial n_j}=\frac{A_{Tj}}{T}-\frac{T}{2}\frac{D_j}{n_j^2}\left(\frac{2H_v}{P}\sum_{k=j}^{m}D_k+H_{bj}-H_v\right) \qquad (10)$$

Because the TRC is convex in T, nj and θ, setting equations (8) and (10) to zero, the optimal out-of-control probability and numbers of shipments for buyer j can be evaluated as:

$$\theta^* = \frac{2iq}{T\left(\sum_{j=1}^{m} D_j\right)^2 g} \tag{11}$$

and

$$n_j^* = T\left[\frac{D_j\left(\frac{2H_v}{P}\sum_{k=j}^{m} D_k + H_{bj} - H_v\right)}{2A_{Tj}}\right]^{\frac{1}{2}} \tag{12}$$

The value of n_j^* should be an integer number, if the calculated n_j^* is not an integer, take the adjacent integer value corresponding to which total cost is minimum. Substituting equation (12) into equation (8) and setting the revised equation to zero, the optimal ordering/production cycle time can be evaluated as:

$$T^* = \left[\frac{2\left(S + \sum_{j}^{m} A_j\right)}{\frac{H_v}{P}\sum_{j=1}^{m} D_j\left(P - \sum_{j=1}^{m} D_j\right) + g\theta\left(\sum_{j=1}^{m} D_j\right)^2}\right]^{\frac{1}{2}} \tag{13}$$

The proof for convexity of TRC on T, n_j and θ is included in Appendix.

Restricted by the second constraint, one or more shipment numbers calculated form equation (12) may be invalid and those shipment numbers should be recalculated. In this kind of case, we suggest that the first buyer's shipment number is reduced to the largest integer that complies with the constraint. Based on $n_1 \geq n_2 \geq \ldots \geq n_m$, the remaining buyers' shipment numbers would not be altered or increased correspondingly.

The following procedure is followed to find optimal values of ordering /production cycle time, shipment numbers and defective probability for the problem under investigation.

Step 1. Set $\theta = \theta_0$ and perform (i)—(iii):

  (i) Substitute θ into equation (13) to find T.
  (ii) Use T to determine θ from equation (11).
  (iii) Repeat (i)—(ii) until no change occurs in the values of T and θ. Denote these solutions by T^* and θ^*, respectively.

Step 2. If $\theta^* \leq \theta_0$, then the solution found in step 1 is optimal for the system and go to step 4.

Step 3. If $\theta^* > \theta_0$, set $\theta^* = \theta_0$ and substitute θ^* into equation (13) to compute T^*.

Step 4. With a given number of buyers, perform (i)—(iii) to determine the order for each buyer:
  (i) Set each buyer be the first buyer in advance, and substitute T^* into equation (12) to calculate n_1 of each buyer.
  (ii) Compare the value of n_1 of each buyer, and assign

the largest one to be the real first buyer.
  (iii) Consider remaining buyers, set each of them to be the j th buyer ($j = 2, 3, \ldots, m$) and use equation (12) to compute n_j, then compare the value of n_j and assign the largest one be the real j th buyer and so on until everyone is allocated a position. Denote these solutions by n_j^*, for $j = 1, 2, \ldots, m$, respectively.

Step 5. If all n_j^* satisfy the constraints, then those shipment numbers found in step 4 are optimal for associated buyers.

Step 6. If one or more n_j^* do not satisfy the constraints, then find new n_j^* through a numerical procedure to minimize equation (7).

V. NUMERICAL ILLUSTRATION

To illustrate the solution procedure presented above, consider an inventory item with the following related parameters tabulated in Table 1. For comparison, a model without quality investment will be also considered; the traditional integrated inventory model is given by:

$$TRC = \frac{1}{T}\left[S + \sum_{j=1}^{m}\left(A_j + n_j A_{Tj}\right)\right]$$
$$+ \frac{T}{2}\left\{\frac{H_v}{P}\sum_{j=1}^{m} D_j\left(P - \sum_{j=1}^{m} D_j\right) + g\theta\left(\sum_{j=1}^{m} D_j\right)^2\right.$$
$$\left.+ \sum_{j=1}^{m}\left[\frac{D_j}{n_j}\left(\frac{2H_v}{P}\sum_{k=j}^{m} D_k + H_{bj} - H_V\right)\right]\right\} \tag{14}$$

Subject to
$$n_1, n_2, n_3, \ldots, n_m \geq 1,$$
$$\frac{T}{n_j} \geq \frac{T}{P}\sum_{j=1}^{m}\frac{D_j}{n_j}$$

To obtain the minimum cost ordering/production cycle time and shipment numbers, take the first derivative of TRC, and set it to zero; thus,

$$\frac{\partial TRC}{\partial T} = -\frac{1}{T^2}\left[S + \sum_{j=1}^{m}\left(A_j + n_j A_{Tj}\right)\right] + \frac{1}{2}\left\{\frac{H_v}{P}\sum_{j=1}^{m} D_j\left(P - \sum_{j=1}^{m} D_j\right)\right.$$
$$\left.+ g\theta\left(\sum_{j=1}^{m} D_j\right)^2 + \sum_{j=1}^{m}\left[\frac{D_j}{n_j}\left(\frac{2H_v}{P}\sum_{k=j}^{m} D_k + H_{bj} - H_v\right)\right]\right\} \tag{15}$$

and

$$\frac{\partial TRC}{\partial n_j} = \frac{A_{Tj}}{T} - \frac{T}{2}\frac{D_j}{n_j^2}\left(\frac{2H_v}{P}\sum_{k=j}^{m} D_k + H_{bj} - H_v\right) \tag{16}$$

The function TRC is convex in T and nj, therefore, solving for the optimal ordering/production cycle time in equation (15) and optimal shipment numbers in equation (16), we obtain

$$n_j^* = T \left[\frac{D_j \left(\frac{2H_v}{P} \sum_{k=j}^{m} D_k + H_{bj} - H_v \right)}{2A_{Tj}} \right]^{\frac{1}{2}} \quad (17)$$

and

$$T^* = \left[\frac{2 \left(S + \sum_{j}^{m} A_j \right)}{\frac{H_v}{P} \sum_{j=1}^{m} D_j \left(P - \sum_{j=1}^{m} D_j \right) + g\theta \left(\sum_{j=1}^{m} D_j \right)^2} \right]^{\frac{1}{2}} \quad (18)$$

Example 1. Suppose there is only one buyer, that is, buyer A. Applying the proposed procedure to cope with one buyer, the optimal solutions are contained in Table 2. From the results in Table 2, the total relevant cost has been reduced by the effects of quality improvement in spite of the holding cost of the traditional model being higher than the proposed models.

Table 1. Data of the example problem.

Buyer	A	B	C
Transportation cost, A_{Tj} (\$)	30	30	20
Holding cost, H_{bj} (\$)	8	8	8
Demand rate, D_j (unit/unit time)	1000	1300	1700
Ordering cost, A_j (\$)	100	100	80
Vendor			
Production rate, P (unit/unit time)		5500	
Setup cost, S (\$)		200	
Holding cost, H_v (\$)		4	
Rework cost, g (\$)		15	
Interest rate, i (\$/unit time)		0.1	
$iq(\theta_0/\theta)$		$400 \ln(0.0002/\theta)$	

Table 2. Optimal solutions for example 1

m = 1	T^* (unit time)	θ^*	n_1^* (unit)	TRC (\$)	Savings (%)
Traditional model	0.31	0.0002	3	2512.17	
Proposed model	0.42	0.0000128166	4	2123.87	15.46%

Example 2. Taking additional buyer B into account, the results of the one-vendor two-buyer partnership are summarized in Table 3. In this circumstance, the integrated setup cost, transport cost, rework cost and holding cost of the proposed model are all lower than those of the traditional model, due to the improved imperfect probability. The influence of quality improvement generates a higher ordering/production cycle time, shipment numbers and lower average inventory for the buyers.

Table 3. Optimal solutions for example 2

m = 2	T^* (unit time)	θ^*	n_1^* n_2^* (unit)	TRC (\$)	Savings (%)
Traditional model	0.19	0.0002	2 2	5466.78	
Proposed model	0.38	0.0000026588	5 4	3615.23	33.87%

Example 3. All buyers listed in Table 1 will be considered now. Again, apply the procedure to deal with a one-vendor three-buyer problem. The solutions are illustrated in Table 4. From the results in Table 4, the savings are very obvious; there is a reduction approaching 52% of total relevant cost in this case. Because, when the demand increases, the vendor's production quantity relatively goes up. If a higher defect rate occurs in the process, it will make both the vendor and the buyer suffer considerable damage to deal with the imperfect items. Furthermore, in the proposed model, the closer the relationship between demand rate and production rate the better the optimal θ which can be reached. Hence, it is worthwhile to invest for quality improvement if there are a large number of demands, especially in the case of supply and demand getting on for balance if assumption 2 is satisfied.

Table 4. Optimal solutions for example 3

m = 3	T^* (unit time)	θ^*	n_1^*	n_2^*	n_3^* (unit)	TRC (\$)	Savings (%)
Traditional model	0.14	0.0002	2	2	1	9307.69	
Proposed model	0.46	0.0000007247	8	6	4	4471.47	51.96%

VI. CONCLUSION

This paper establishes an integrated inventory model based on previous work of Siajadi et al [19] to investigate the ordering/production cycle time, the numbers of deliveries and quality in the one-vendor multiple-buyer purchasing partnership and an imperfect process environment. The objective is to minimize the total relevant cost per unit time through small lot size, frequent delivery and investment for quality improvement, and adopting classical optimization techniques to simultaneously optimize ordering/production cycle time, shipment numbers and process quality. A procedure for finding the optimal solution is developed, and three numerical examples are given to illustrate the advantages of this model.

The return on investment for quality improvement is substantial and many papers have shown that improving quality could reduce waste, in other words, cut the cost. In addition, the probability of defects also makes a great impact on the inventory policy regarding production cycle and lot size, so it is important to always take quality issues into account for any business in a competitive supply chain environment nowadays.

APPENDIX

The Hessian matrix H of TRC can be obtained as follow:

$$\mathbf{H}=\begin{bmatrix} \dfrac{\partial^2 TRC}{\partial T^2} & \dfrac{\partial^2 TRC}{\partial T \partial \theta} & \dfrac{\partial^2 TRC}{\partial T \partial n_j} \\[2mm] \dfrac{\partial^2 TRC}{\partial \theta \partial T} & \dfrac{\partial^2 TRC}{\partial \theta^2} & \dfrac{\partial^2 TRC}{\partial \theta \partial n_j} \\[2mm] \dfrac{\partial^2 TRC}{\partial n_j \partial T} & \dfrac{\partial^2 TRC}{\partial n_j \partial \theta} & \dfrac{\partial^2 TRC}{\partial n_j^{2}} \end{bmatrix} \tag{A.1}$$

Where

$$\frac{\partial^2 TRC}{\partial T^2} = \frac{2}{T^3}\left[S + \sum_{j=1}^{m}\left(A_j + n_j A_{Tj}\right)\right]$$

$$\frac{\partial^2 TRC}{\partial n_j^{2}} = \frac{T D_j}{n_j^{3}}\left(\frac{2H_v}{P}\sum_{k=j}^{m} D_k + H_{bj} - H_v \right)$$

$$\frac{\partial^2 TRC}{\partial \theta^2} = \frac{iq}{\theta^2}$$

$$\frac{\partial^2 TRC}{\partial \theta \partial T} = \frac{\partial^2 TRC}{\partial T \partial \theta} = \frac{\left(\sum_{j=1}^{m} D_j\right)^2 g}{2}$$

$$\frac{\partial^2 TRC}{\partial n_j \partial T} = \frac{\partial^2 TRC}{\partial T \partial n_j}$$

$$= -\frac{A_{Tj}}{T^2} - \frac{1}{2}\frac{D_j}{n_j^2}\left(\frac{2H_v}{P}\sum_{k=j}^{m} D_k + H_{bj} - H_v \right)$$

$$\frac{\partial^2 TRC}{\partial \theta \partial n_j} = \frac{\partial^2 TRC}{\partial n_j \partial \theta} = 0 .$$

Next, evaluate the principal minor of H at optimal solution. The first principal minor of H is

$$\left|H_{11}\right| = \frac{\partial^2 TRC}{\partial T^2} = \frac{2}{T^3}\left[S + \sum_{j=1}^{m}\left(A_j + n_j A_{Tj}\right)\right] > 0. \tag{A.2}$$

Since from equation (A.1), the second principal minor of H is:

$$\left|H_{22}\right| = \left[\frac{\left(\sum_{j=1}^{m} D_j\right)^2 g}{2}\right]^2$$

$$\times \left\{ \frac{T\dfrac{H_v}{P}\sum_{j=1}^{m} D_j\left(P - \sum_{j=1}^{m} D_j\right)}{iq} \right.$$

$$\left. + \frac{T\sum_{j=1}^{m}\dfrac{D_j}{n_j}\left(\dfrac{2H_v}{p}\sum_{k=j}^{m} D_k + H_{bj} - H_V\right)}{iq} + 1 \right\} > 0 \tag{A.3}$$

The third principal minor of H is:

$$\left|H_{33}\right| = \frac{2}{\left(T^*\right)^3}\left[S + \sum_{j=1}^{m}\left(A_j + n_j^{*} A_{Tj}\right)\right]$$

$$\times \frac{T^* D_j}{\left(n_j^{*}\right)^3}\left(\frac{2H_v}{P}\sum_{k=j}^{m} D_k + H_{bj} - H_v \right) \times \frac{iq}{\left(\theta^*\right)^2}$$

$$-\left(-\frac{A_{Tj}}{\left(T^*\right)^2} - \frac{1}{2}\frac{D_j}{\left(n_j^{*}\right)^2}\left(\frac{2H_v}{P}\sum_{k=j}^{m} D_k + H_{bj} - H_v\right)\right)^2 \times \frac{iq}{\left(\theta^*\right)^2}$$

$$-\left[\frac{\left(\sum_{j=1}^{m} D_j\right)^2 g}{2}\right]^2 \times \frac{T^* D_j}{\left(n_j^{*}\right)^3}\left(\frac{2H_v}{P}\sum_{k=j}^{m} D_k + H_{bj} - H_v\right)$$

$$=\left\{ \left[\frac{\left(\sum_{j=1}^{m} D_j\right)^2 g}{2}\right]^2 \times \frac{2 A_{Tj}}{T^* n_j^{*}} \right\} \times \left\{ \frac{2\left[S + \sum_{j=1}^{m}\left(A_j + n_j A_{Tj}\right)\right]}{T^* iq} - 1\right\}$$

$$-\frac{A_{Tj}^2\left(\sum_{j=1}^{m} D_j\right)^4 g^2}{\left(T^*\right)^2 iq}$$

$$=\left\{ \left[\frac{\left(\sum_{j=1}^{m} D_j\right)^2 g}{2}\right]^2 \times \frac{2 A_{Tj}}{T^* n_j^{*}} \right\}$$

$$\times \left\{ \frac{T^*\left[\dfrac{H_v}{P}\sum_{j=1}^{m} D_j\left(P - \sum_{j=1}^{m} D_j\right) + \sum_{j=1}^{m}\dfrac{2 n_j^{*} A_{Tj}}{\left(T^*\right)^2}\right]}{iq} + 1 \right\}$$

$$-\frac{A_{Tj}^2\left(\sum_{j=1}^{m} D_j\right)^4 g^2}{\left(T^*\right)^2 iq}$$

$$=\left(\frac{A_{Tj}^2\left(\sum_{j=1}^{m} D_j\right)^4 g^2}{\left(T^*\right)^2 iq}\right)$$

$$\times \left\{ \frac{T^* iq}{2 n_j^{*} A_{Tj}}\left(\frac{T^*\left[\dfrac{H_v}{P}\sum_{j=1}^{m} D_j\left(P - \sum_{j=1}^{m} D_j\right) + \sum_{j=1}^{m}\dfrac{2 n_j^{*} A_{Tj}}{\left(T^*\right)^2}\right]}{iq} + 1 \right) - 1\right\}$$

$$= \left(\frac{A_{Tj}^2 \left(\sum_{j=1}^{m} D_j \right)^4 g^2}{\left(T^* \right)^2 iq} \right) \quad (A.4)$$

$$\times \left\{ T^* \left(\frac{\frac{T^* H_v}{P} \sum_{j=1}^{m} D_j \left(P - \sum_{j=1}^{m} D_j \right) + iq}{2n_j^* A_{Tj}} \right) + \left(\frac{\sum_{j=1}^{m} 2n_j^* A_{Tj}}{2n_j^* A_{Tj}} - 1 \right) \right\} > 0$$

$\sum_{j=1}^{m} 2n_j^* A_{Tj} / 2n_j^* A_{Tj}$ is always greater than 1, hence, we have $|H_{33}| > 0$.

Consequently, the results from (A.2) to (A.4) show that the Hessian matrix is positive and *TRC* is convex with respect to *T*, θ and n_j.

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