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SEMI-ACTIVE VIBRATION CONTROL FOR OFFSHORE PLATFORMS BASED ON LQG METHOD

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Key words**:** semi-active control, LQG method, offshore platform, MR damper.

ABSTRACT

The objective of this paper is to mitigate the wave-induced vibration of offshore platform with magneto-rheological (MR) damper. The model of the platform coupled with MR damper is established where the external wave force is approximated with a white noise via a designed filter. Based on Linear Quadratic Gaussian (LQG) method, the optimal control force is determined when taking the measurement noise into account. Semi-active control algorithm is applied to generate the MR damping force by comparing with the optimal control force. Numerical example demonstrates that the semi-active control strategy based on LQG method can reduce the responses of the platform effectively.

I. INTRODUCTION

Offshore platforms, which work in hostile sea environment, are continuously exposed to environmental loads such as wind, waves, current and may undergo continuous vibration. The vibration, on one hand, will cause fatigue damage, decreasing the platform's reliability; on the other hand, the excessive vibration can't satisfy the basic psychological requirements of the personnel. Hence it is essentially important to mitigate the vibration response of offshore platforms. Structural vibration control of offshore structures under wave loading has drawn much attention from designers and researchers, which has become a very important research subject in ocean engineering and academic fields [1, 5].

Generally speaking, structural control can be divided into four categories [2-5, 7, 8, 11, 14, 15]: passive control, active control, semi-active control and hybrid control. Each of these methods has distinct advantages and disadvantages that will determine the appropriateness of any of the methods for a particular application. A semi-active control system may be defined as a system which utilizes the response of the structure to generate the control force, which is passive, but can be adjusted by the external power source. Therefore, semi-active control systems possess advantages of passive and active control systems, which include high reliability, good control effect and smaller external energy requirements. Among the semi-active control devices, magneto-rheological (MR) damper is used widely in structure vibration control. This device has many attractive characteristics including quick reaction with little time delay, insensitivity to temperature, small power requirement, high reliability and stability. A number of researches on it have been performed on numerical studies and experimental applications. Li et al. [12] demonstrated that the MR damper with optimal control theory can significantly reduce the maximum responses and the root-mean-square values. Zhou et al. [16] proposed a semi-active control method utilizing energy dissipation principle and bang-bang control based on LQR optimal control theory. Kawano [6] investigated semi-active control devices applied in jacket offshore platform. The active control force can be determined by time-domain transient optimal control method.

As described above, a lot of control strategies have been investigated and illustrated to be effective for structural vibration mitigation. However there are still a number of challenges ahead. For example, full state feedback is required for an effective control which will need a lot of sensors, and uncertainty in measuring state variables can't be considered. In this paper, the LQG regulator is designed to calculate the desired optimal control force. It requires only acceleration response as feedback which can be measured more easily. Meanwhile, this strategy is able to take the measurement noise into consideration. Numerical simulation is conducted of an offshore platform under wave excitation, where MR damper is coupled in the offshore platform to generate semi-active force. Results demonstrate that the present semi-active control strategy based on LQG method can reduce the responses of the platform effectively.

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II. ESTABLISHMENT OF THE PLATFORM AND CONTROL SYSTEM

1. Formulation of motion equation of platform coupled with MR damper

The MR damper used in semi-active control of platform is generally installed as a damping element, supposing that the stiffness of it can be omitted. Since structural responses are dominated mainly by the first mode of system, the offshore platform is modeled as a linear single degree of freedom (SDOF) structure by extracting its first vibration mode. The model of the platform coupled with the MR damper is shown in Fig.1

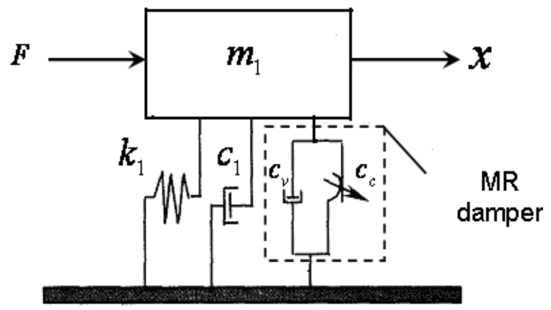

Fig.1. Schematic diagram of a SDOF platform coupled with a MR damper

According to Fig.1, the governing equation of platform-MR damper can be written as:

$$m_1\ddot{x} + c_1\dot{x} + k_1 x = F + F_d \tag{1}$$

where m_1, c_1 and k_1 are the first modal mass, damping and stiffness of the platform respectively; F denotes the random wave force acting on the platform; F_d means the control force generated by MR damper; x is the displacement of the platform; dot denotes differentiation with respect to time t. Defining a state vector, $X = [x, \dot{x}]^T$, Eq. (1) can be expressed in state space as follows:

$$\dot{X} = AX + BF_d + HF \tag{2}$$

where A, B and H are determined by structural parameters which can be expressed as follows:

$$A = \begin{bmatrix} 0 & 1 \\ -\dfrac{k_1}{m_1} & -\dfrac{c_1}{m_1} \end{bmatrix}, B = \begin{bmatrix} 0 \\ \dfrac{1}{m_1} \end{bmatrix}, H = \begin{bmatrix} 0 \\ \dfrac{1}{m_1} \end{bmatrix} \tag{3}$$

In this paper acceleration is adopted as output. The output equation can be written as:

$$y = C_a X + D_a F_d + H_a F \tag{4}$$

where C_a, D_a and H_a can be expressed as:

$$C_a = \begin{bmatrix} -\dfrac{k_1}{m_1}, & -\dfrac{c_1}{m_1} \end{bmatrix}, D_a = H_a = [\frac{1}{m_1}] \tag{5}$$

2. Determination of the random wave force

The random wave force *F* acting on the platform can be formulated through the linear Morison equation for an input Power Spectral Density (PSD) of wave elevation [10]. The generalized wave force is expressed in a generalized wave force spectrum

$$S_F(\omega) = \left|T_{F\eta}(\omega)\right|^2 S_\eta(\omega) \tag{6}$$

where $S_\eta(\omega)$ is the wave elevation spectrum and $T_{F\eta}(\omega)$ is the transfer function from wave elevation to wave force, which can be determined according to the platform structure .

Since a white noise process is needed as the input in an optimal control system, a shaping filter with an input of white noise *w* and an output of wave force *F* is designed based on the following two requirements: i) the input is unit white noise process. ii) the PSD of the output denoted as $\hat{S}_F(\omega)$ must be as close to $S_F(\omega)$ given in Eq. (6) as possible. It can be estimated by way of spectral factorization. For numerical computation, it is assumed that this estimated wave force spectrum takes the following form [9]:

$$\hat{S}_F(\omega) = \frac{B(\frac{\omega}{\omega_0})^4}{(\frac{\omega}{\omega_0})^8 - \alpha_1(\frac{\omega}{\omega_0})^6 + \alpha_2(\frac{\omega}{\omega_0})^4 - \alpha_3(\frac{\omega}{\omega_0})^2 + \alpha_4} \tag{7}$$

where ω_0 is the peak frequency.

By use of the least square fitting method, the coefficients B, α_1, α_2, α_3 and α_4 can be determined. The equation can be written in transfer function from a unit intensity white noise to the generalized force:

$$\hat{S}_F(\omega) = \left|T_{Fw}(\omega)\right|^2 S_0 \tag{8}$$

Using a state space representation, the transfer function can be expressed as follows:

$$\begin{cases} \dot{X}_w(t) = A_w X_w(t) + B_w w(t) \\ F(t) = C_w X_w(t) \end{cases} \tag{9}$$

where X_w is a 4 by 1 vector; A_w, B_w and C_w are 4 by 4, 4 by 1 and 1 by 4 coefficient matrices respectively; $w(t)$ is a zero mean unit intensity white noise process. In the following study, Eq. (9) can be used as the wave force acting on the platform.

Substituting Eq. (9) into Eqs. (2) and (4), the joint state-space equation about platform-MR damper-shaping filter can be obtained as follows:

$$\begin{cases} \dot{Z} = A_z Z + B_z F_d + H_z w \\ y = C_z Z + D_z F_d + v \end{cases} \tag{10}$$

where $Z = \begin{bmatrix} X \\ X_w \end{bmatrix}$ is an expanded state vector. A_z, B_z, G_z and C_z, D_z can be expressed as follows:

$$A_z = \begin{bmatrix} A & HC_w \\ 0 & A_w \end{bmatrix}, B_z = \begin{bmatrix} B \\ 0 \end{bmatrix}, H_z = \begin{bmatrix} 0 \\ B_w \end{bmatrix},$$

$$C_z = \begin{bmatrix} C_a & H_a C_w \end{bmatrix}, D_z = D_a = \begin{bmatrix} \frac{1}{m_1} \end{bmatrix} \tag{11}$$

III. LQG CONTROL STRATEGY

LQG **(**Linear Quadratic Gaussian**)** control problem is one the most fundamental optimal control problem. Compared to the traditional LQ controller, it concerns uncertain linear systems disturbed by white Gaussian noise, having incomplete state information (i.e., not all the state variables are measured and available for feedback) and undergoing control subject to quadratic costs. Moreover, LQG can consider a more complex situation where random disturbances exist in measuring state variables and output variables. A detailed LQG control block diagram is shown in Fig.2. As indicated, the LQG controller is composed of a Kalman estimator with a linear-quadratic regulator.

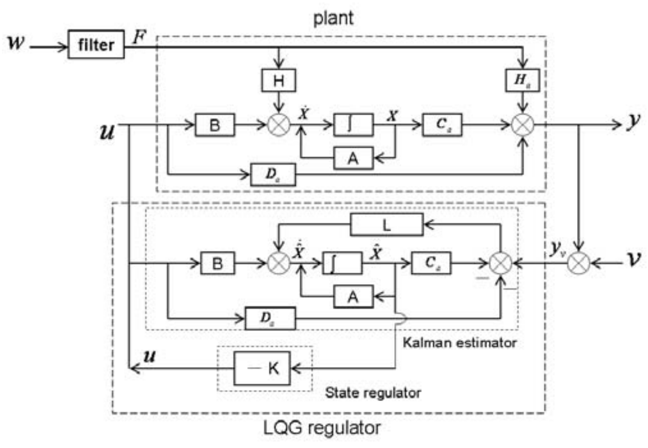

Fig.2 Block diagram of LQG control system

In Fig.2, the plant is the offshore platform which is simplified to be a SDOF system. State space representation of the plant is expressed as follows:

$$\begin{cases} \dot{X} = AX + Bu + HF \\ y = C_a X + D_a u + H_a F \end{cases} \tag{12}$$

where F is the random wave force acting on the platform and can be determined with a unit intensity white noise $w(t)$ via a designed filter, as indicated in Eq. (9). u is the optimal control input calculated by LQG control method. Acceleration response y is adopted as the output, since it can be measured easily. A, B and H are shown in Eq. (3) and C_a, D_a, H_a can be expressed in Eq. (5).

LQG method generates optimal force by minimizing the following quadratic performance index function:

$$J = E\left\{\int_0^{\infty} \left[X^T(t)QX(t) + u^T(t)Ru(t)\right]dt\right\} \tag{13}$$

where Q and R are symmetric semi-definite and positive definite weight matrices, respectively. The choice of Q and R is a tradeoff between the magnitude of the response and the cost of control force.

1. State regulator

As described above, the LQG regulator is composed of two parts. The first part is the state regulator, which is designed to calculate the optimal gain matrix K using the following feedback law for minimizing the objective function J.

$$u(t) = -KX(t) \tag{14}$$

The designed optimal gain matrix of state-feedback K under the performance index (Eq. (13)) yields:

$$K = R^{-1}B^T P \tag{15}$$

where P is the solution of Riccati equation:

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \tag{16}$$

2. Kalman estimator

Because optimal control of Eq. (14) can only be realized with complete state vector, a state estimate $\hat{X}$ is constructed to replace state X, by minimizing the expectation of steady-state error covariance $\lim_{t\to\infty} E\left\{(X-\hat{X})(X-\hat{X})^T\right\}$. State estimate $\hat{X}$ can be generated from using kalman estimator:

$$\begin{aligned} \dot{\hat{X}}(t) &= A\hat{X}(t) + Bu(t) + L(y_v(t) - C_a\hat{X}(t) - D_a u(t)) \\ &= (A - LC_a)\hat{X}(t) + (B - LD_a)u(t) + Ly_v(t) \end{aligned} \tag{17}$$

In Eq.(17), y_v is the output considering the measurement uncertainties, indicated as follows:

$$y_v(t) = y(t) + v(t) \tag{18}$$

where $y(t)$ is the dynamic response of the plant. In the measuring process, random disturbances maybe influence the measurement accuracy. $v(t)$ is indicative of the random noise coming from measurement and is assumed to be an irrelevant Gaussian white noise process, satisfying $E(v)=0$ and $E(vv^T)=R_0$.

The filter gain L is determined by extracting the first and second elements of the vector L_1 as follows:

$$L_1 = P_1 C_z^{\,T} R_0 \tag{19}$$

P_1 is the solution of Riccati equation:

$$A_z P_1 + P_1 A_z^{\,T} - P_1 C_z^{\,T} R_0^{-1} C_z P_1 + H_z Q_0 H_z^{\,T} = 0 \tag{20}$$

In summary, LQG regulator can be formed given state-feedback gain K and Kalman estimator. The state space representation of the LQG regulator can be expressed as follows:

$$\begin{cases} \dot{\hat{X}} = \left[A - LC_a - (B - LD_a)K\right]\hat{X} + Ly_v \\ u = -K\hat{X} \end{cases} \tag{21}$$

So the optimal control force $u(t) = -K\hat{x}(t)$ can be obtained from LQG regulator.

IV. MR DAMPER DESIGN

1. Mechanical model of MR damper

MR damper is a semi-active control device that uses magneto-rheological fluids to yield adjustable damping force. When the magnetic field intensity increases, MR fluids change from viscous fluids to yielding viscoplastic solids within milliseconds. The constitutive relation is usually depicted as the Bingham model of viscoplasticity:

$$\tau = \tau_y \operatorname{sgn}(\dot{\gamma}) + \eta\dot{\gamma} \tag{22}$$

where τ is the applied shear stress; τ_y is the yield stress; η is the viscosity coefficient; $\dot{\gamma}$ is shear strain velocity. According to Phillips and Makris's derivation, Ou and Guan [13] derived the following simplified model of MR damper' resilience:

$$F_d(t) = \frac{12\eta L A_p}{\pi D h^3} A_p \dot{x}(t) + \frac{3L\tau_y}{h} A_p \operatorname{sgn}[\dot{x}(t)] + f \tag{23}$$

where L is the axial piston length; D is the diameter of the cylindrical tank; h is the gap between the piston and cylinder; A_p is the cross-sectional area of the piston; $\dot{x}(t)$ is the relative velocity of the piston to the cylinder; f is the friction force in the damper, which is omitted in this paper. The damping force can be rewriten in the following form:

$$F_d(t) = c_v \dot{x}(t) + f_c \operatorname{sgn}(\dot{x}(t)) \tag{24}$$

where c_v is the viscous damping coefficient which is a constant determined by damper's parameters; f_c is the coulomb damping force which can be adjusted by magnetic field. The main parameters of the MR damper used in this paper are tabulated in Table 1.

Table 1. Parameters of the MR damper

D mm	d mm	h mm	L mm	η $Pa \cdot s$	$\tau_{y\max}$ kPa
100	60	2	60	0.6	40

2. Algorithm for semi-active control

The active force calculated by LQG method is optimal. However, the MR damper can only produce output force passively and the force which could be produed is related to the parameters of the MR damper. Therefore, the practical control force generated by MR damper can't equal the active optimal control force simultaneously. It is expected that the desired output force of MR damper is as close as possible to the optimal force by LQG method through adjustment of the magnetic field. Thus a semi-active control strategy associated with MR damper can be depicted as follows:

$$F_d(t) = \begin{cases} F_{d\max} & F_d \cdot u > 0 \ \& \ F_{d\max} < |u| \\ u(t) & F_d \cdot u > 0 \ \& \ |u| \in [F_{d\min}, F_{d\max}] \\ F_{d\min} & others \end{cases} \tag{25}$$

where u is the desired optimal active force calculated by LQG method; $F_d(t)$ is the force generated by MR damper which can be adjusted by magnetic field; $F_{d\max}$ and $F_{d\min}$ are the maximum and minimum force produced by MR damper corresponding to the maximum and minimum magnetic field respectively.

V. NUMERICAL EXAMPLE

1. Features parameters of the platform

In this section, the semi-active control approach based on LQG method is applied to a realistic offshore platform [12] which located in Bohai Bay. The structure comprises a jacket template, foundation pile and two level deck systems. The main parameters for the platform are shown in Table 2.

2. Sea state

The sea state herein is characterized by using the Jonswap wave spectrum. The generalized wave force acting on the platform can be formulated through the linear Morison equation for an input Power Spectral Density (PSD) of wave elevation. The corresponding parameters are listed in Table 3.

Table 2. Main parameters for the platform

Total mass (kg)	2708900
First modal mass(kg)	2371100
Fundamental frequency(Hz)	0.35
Structural damping ratio	0.04
Diameter of the legs(m)	1.70
Total height(m)	43.1
Number of legs	4

Table 3. Parameters for the wave loading

Dominant wave period T_s (sec)	4.5
Significant wave height H_s (m)	2.5
Sharpness magnification factor γ	3.3
Water depth d(m)	13.2
Inertial coefficient C_m	2.0
Drag coefficient C_d	1.5

Since a white noise process is needed as the input in LQG control algorithm, a designed filter with an input of white noise w and an output of wave force can be estimated by way of spectral factorization. The power spectral density of wave elevation and wave force are shown in Fig.3 and Fig.4 respectively. In Fig.4, the curves illustrate that the filtered spectrum is a relatively good approximation of the target theoretical spectrum.

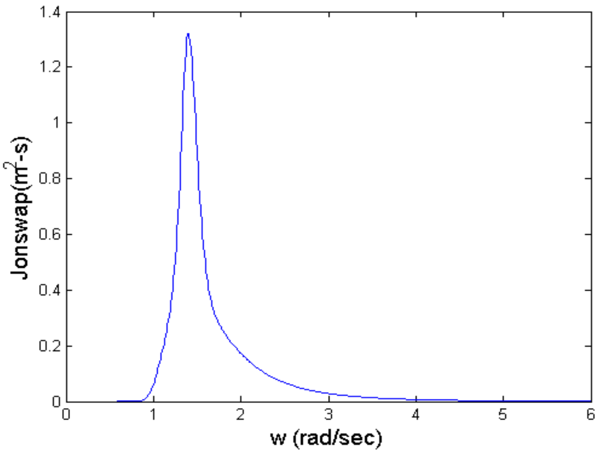

Fig.3 PSD of the wave elevation

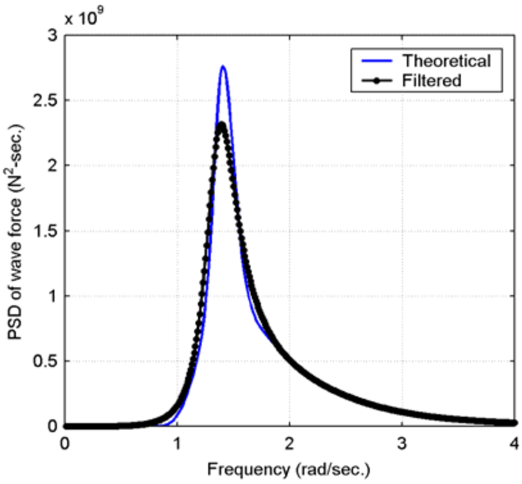

Fig.4 PSD of the wave force

3. Comparison of the optimal control force and MR damper force

In this paper, the weight matrices of Q, R and covariance of Q_0, R_0 are chosen as:

$$Q = \begin{bmatrix} 10^7 & 0 \\ 0 & 10^7 \end{bmatrix},\ R = 10^{-6}, Q_0 = 1, R_0 = 0.1$$

The optimal control force u is calculated by the LQG method which is an active control strategy. Therefore, u is the desired optimal active force. It is shown in Fig. 5 in dotted curve. However, practical control force F_d produced by MR damper is semi-active. It is denoted in Fig.5 with solid curve. Shown also in Fig.5 is the MR damper force via semi-active control algorithm. It can be seen from Fig.5 that since MR force is limited to the parameters of the MR damper, the control force generated by MR damper is unable to equal the desired optimal force simultaneously.

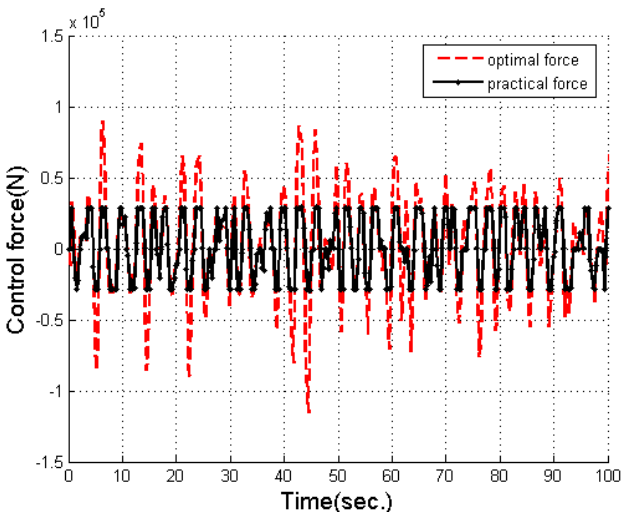

Fig.5 MR damper force compared with optimal force

4. Control performance of MR damper based on LQG method

The structural vibration control with MR damper based on LQG method is investigated. The displacement and acceleration of the platform with and without the MR damper are depicted in Figs.6 and 7, respectively. From these figures, it is evident that the MR damper based on LQG method can effectively reduce the response of the platform. The maximum and root mean square (RMS) values of displacement and acceleration and their control performances are tabulated in Table 4. As is seen from Table 4, the maximum displacement and acceleration of platform are reduced by 46.89% and 49.73%, respectively. The root mean square values of displacement and acceleration are reduced by 52.33% and 56.82%, respectively. The MR damper with the LQG controller can reduce the structural vibration significantly.

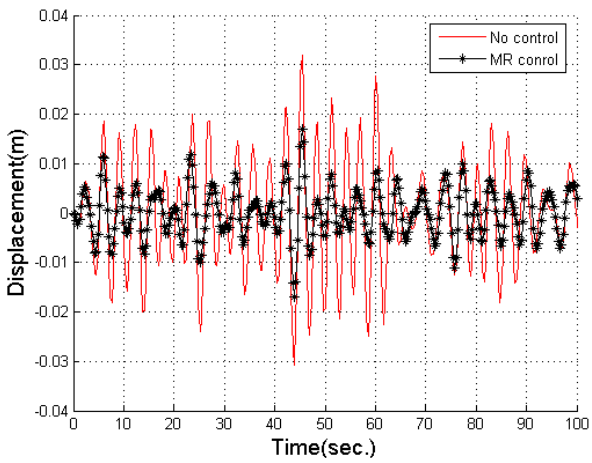

Fig.6 Displacements of the platform

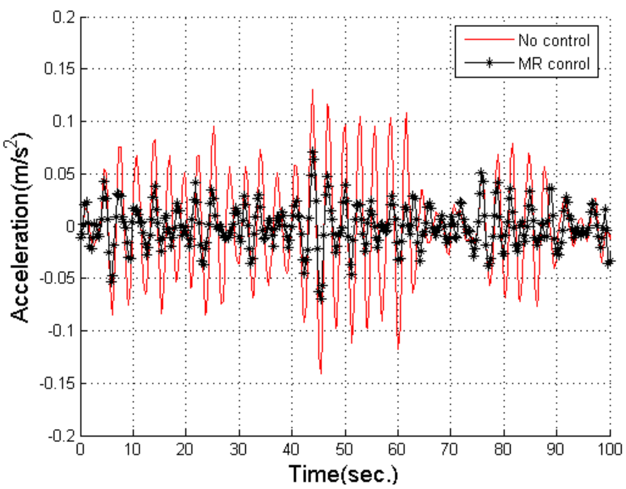

Fig.7 Acceleration of the platform

Table 4. Responses of the platform and the control effect (K_2=0.1)

		No control	With control	Reduction (%)
Displacement (m)	Maximum	0.0320	0.0170	46.89
	RMS	0.0092	0.0044	52.33
Acceleration (m/s^2)	Maximum	0.1406	0.0707	49.73
	RMS	0.0406	0.0175	56.82

For investigating the degree of uncertainty in measuring response, a factor K_2 is introduced, defined as the energy intensity ratio of measurement noise v to input noise w. This coefficient expresses the strength of disturbance's uncertainty in measurement. The larger coefficient K_2, the higher degree of the uncertainty. The control effect for different coefficient K_2 is given in Table 5. It can be seen that the LQG control algorithm is relatively not sensitive to the measurement uncertainties. When the energy intensity ratio K_2 changes from 0.1 to 0.6, the RMS reduction of displacement changes only from 52.33% to 46.06%, and acceleration reduction changes from 56.82% to 47.98%. In other words, the control system has better robustness.

Table 5. Control effect influenced by measurement noise

K_2	Reduction of RMS (%) displacement response	Reduction of RMS (%) Acceleration response
0.1	52.33	56.82
0.2	51.67	55.93
0.3	49.95	53.51
0.6	46.06	47.98
1.0	43.85	44.82

VI. CONCLUSION

In this paper, the effectiveness of semi-active vibration control of a steel jacket platform associated with MR damper based on LQG method is investigated by numerical simulation. The calculation and analysis show that: (1) The semi-active control with the MR damper can effectively reduce the response of the offshore platform due to random wave force. (2) The semi-active control system based on LQG method only needs structural acceleration as state feedback, making the whole system more easily applicable. (3) LQG design is able to take the measurement noise into account, and the control system has better robustness.

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