METHODS FOR DESIGNING PARTIALLY INFLATED GEOTUBES

Shu-Wang Yan, Jing Chen, and Li-Qiang Sun

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ABSTRACT

A widely used technique in constructing dikes for land reclamation is to use tubes made of sewn geosynthetic sheets. These tubes are usually filled with slurry comprising soils, such as sand, silt, and clay. The tension stress that develops in geosynthetic tubes during tube filling is the dominant factor considered in constructing safe dikes. Existing design methods are effective for sausage-shaped tube designs; however, they cannot be directly applied in flat tubes, which are commonly used in dike construction. This paper presents a procedure that can determine the relationships among tube size, pumping pressure, unit weight of the slurry, and tension stress in geosynthetic tubes. All these approaches are programmed to enable dike designers to select a suitable geosynthetic design for dike profiles.

I. INTRODUCTION

The technique of using tubes made of geosynthetic sheets to construct dikes for land reclamation has been widely employed in many coastal areas in China and elsewhere in the world (Bogossian et al., 1982; Silverster, 1986; Ockels, 1991; Pilarczyk, 1994; Yan and Chu, 2010). These tubes can be filled with local soils with good permeability, such as sand and silt. Thus, the construction cost that this technique entails can be much lower than that required by other dike construction methods.

In designing dikes with geosynthetic tubes, the tension force that develops in these tubes during soil filling and piling up must be determined to ensure the appropriate strength of the material. In practice, tube height must be controlled during soil filling, and specific pump pressure and tube size must be considered. Therefore, the relationships among tube size, pumping pressure, unit weight of the slurry, and tension stress in geosynthetic tubes need to be identified in the design stage. Several design methods have been presented in previous works (e.g., Liu, 1981; Leshchinsky et al., 1992(a), 1992(b); Carroll, 1994; Kazimiemvicz, 1994; Leshchinsky and Leshchinsky, 1995; Chen et al., 2015). These studies provide solutions to cases in which designed tubes are inflated to a certain height (defined as the “perfect height” in this paper). However, tubes used in practical projects are very flat and large and feature a perimeter of more than 25 m. In this case, the tube height is lower than the perfect height. Hence, the typical method cannot be directly used to design these types of geosynthetic tubes. This is a problem which clearly emerges when calculating tensile force and other parameters for partially inflated tubes.

In this study, a procedure is developed to enhance the existing design method. This procedure considers important details in the design of a flat, partially inflated geosynthetic tube to define the shape of the tube and to determine the relationship among tube size, pumping pressure, unit weight of the slurry, and tension stress. This method solves the problem related to partially inflated tubes and facilitates the development of several fitted curves to satisfy the requirements of practical projects. All these approaches are programmed to enable dike designers to select a suitable geosynthetic design for dike profiles.

II. DESIGN OF FULLY INFLATED GEOSYNTHETIC TUBES
1. Procedure for Designing Partially Inflated Tubes

The tubes used for dike construction are usually made of sewn geosynthetic sheets. The inlet openings on top of the tube are for the attachment of a pipe that transports hydraulic fill into the tubes (Fig. 1). The formulation of a geosynthetic tube filled with pressurized slurry or fluid is based on the equilibrium of the encapsulating flexible shell, thus proving that the material of the encapsulating shell features circumferential tensile force and cylindrical geometry. Notably, the formulation appears in numerous articles (e.g., Liu, 1981; Carroll, 1994; Kazimierowicz, 1994; Yan and Chu, 2010; Chu et al., 2011, 2012; Guo et al., 2011; Yan et al., 2015). For brevity and convenience, only an overview of the basic formulation is presented in this paper. Several assumptions listed below govern the formulation.

1) The problem is two-dimensional (2D) (i.e., plane strain) in nature, that is, the tube is a long axis and all cross sections perpendicular are identical in terms of geometry and materials. The pressure at the inlet (i.e., pumping pressure) is an important factor for analysis.

2) The geosynthetic shell is thin and flexible and is subjected to negligible strain during filling.

3) The material filling the tube is slurry (i.e., fluid); therefore, a hydrostatic state of stress exists inside the tube.

4) No shear stress can develop between the slurry and the geosynthetic.

Fig. 2 presents the cross-sectional view of a geosynthetic tube. For clarity of presentation, the tube considered is surrounded by air and filled with a single type of slurry. However, the extension of the formulation to include the layers of slurry inside and the layers of fluid outside is straightforward. The cross section is symmetrical and has a maximum height of \( h \) at the centerline, a maximum width \( B \), and a flat base \( b \), which is in contact with the foundation soil. The pumping pressure of the slurry into the tube is \( p_0 \), and the average density of the slurry is \( \gamma \). Hence, the hydrostatic pressure of the slurry at any depth \( x \), as measured from point \( O \), is

\[
p(x) = p_0 + \gamma x.
\]
The geometry of a geosynthetic shell is defined by an unknown function \( y = f(x) \). At point \( S(x, y) \), the radius of the curvature of the geosynthetic shell is \( \rho \). The center of this curvature is at point \( C(x_c, y_c) \). Both \( \rho \) and \( C \) vary along \( y(x) \). The forces acting on the infinitesimal arc length \( ds \) of the geosynthetic tube at \( S \) (inset in Fig. 2) are considered. The geosynthetic tensile force \( T \) must be constant along the circumference because the problem is assumed to be 2D and no shear stress can develop between the slurry and the geosynthetic shell. Assembling the force equilibrium equation in either the \( x - \) or \( y - \) direction leads to the relationship expressed as

\[
\rho(x) = T / p(x) \tag{1}
\]

Eq. (1) is valid at any point along \( A_1OA_2 \). To simplify the analysis, the calculated \( T \) from Eq. (1) is conservatively assumed to be carried solely by the geosynthetic tube along the flat base \( b \) (i.e., no portion of \( T \) is transferred to the foundation soil because of the shear stress along the interface between the geosynthetic tube and soil; this shear can be mobilized only as the geosynthetic tube deforms relative to the foundation). Consequently, Eq. (1) expresses the complete solution for the problem, and the radius of curvature is computed via differential calculus using the expressions given by

\[
\rho(x) = \frac{[1 + (y')^2]^{3/2}}{y''} \tag{2}
\]

where \( y' = dy/dx \) and \( y'' = d^2y/dx^2 \).

Substituting Eq. (2) and \( p(x) \) into Eq. (1) yields

\[
Ty' - (p_o + \gamma x)[1 + (y')^2]^{3/2} = 0 \tag{3}
\]

Eq. (3) is a nonlinear differential equation that generally has no closed-form solution; thus, it must be numerically solved. Its solution shown below determines the relationships among the tube geometry \( y(x) \), circumferential tensile force \( T \), pumping pressure \( p_o \), unit weight of slurry \( \gamma \), and tube height \( h \) (i.e., \( x \) varies only between zero and \( h \)).

\[
y = f(x \mid T, p_o, h, \gamma) \tag{4}
\]

Given that the unit weight of the slurry \( \gamma \) is known, Eq. (4) implies that \( y \) is a function of the independent variable \( x \) and the three parameters \( T, p_o \), and \( h \). Typically, \( y(x) \) is sought for a given (design) parameter, that is, one of three parameters \( (T, p_o, \text{or} h) \) is given, and the other two parameters are part of the solution to the problem. To obtain this explicit solution, constraints must be imposed. Two such constraints produce a solution in which the tube geometry and the other two parameters can be obtained for a selected design parameter. As such, two physical constraints can be used to replace the two unknown parameters, which are currently part of the solution.

One constraint is the geometrical boundary condition at point \( O \). Physically, the geosynthetic tube at \( O \) must be horizontal to ensure the smooth transition from one-half of the tube of the symmetrical problem to the other half. The expression is given by

\[
1 / y'(0) = 0 \tag{5}
\]

The second constraint can be introduced through the specification of the flat base length \( b \). In this case, the vertical force equilibrium along \( b \) requires

\[
b = W / (p_o + \gamma h) \tag{6a}
\]

where \( W \) is the weight per unit length of the slurry filling the entire section of the tube and

\[
W = 2\gamma \int_0^h y(x)dx \tag{6b}
\]

Combining Eqs. (6a) and (6b) leads to the following equation:

\[
b = \frac{2\gamma}{p_o + \gamma h} \int_0^h y(x)dx \tag{7}
\]

Prescribing \( b \) and simultaneously solving Eqs. (3), (5), and (7) for a single selected design parameter (any of \( T, p_o \), or \( h \)) can result in a tube with a certain length of circumference \( L \). However, a tube is preferred over \( b \) because the tube is manufactured from a prescribed number of geosynthetic sheets sewn together. If \( L \) is specified, the value of \( b \) becomes the outcome of the analysis. Hence, Eq. (7) can be replaced with the constraint given by

\[
L = b + \int_s ds \tag{8}
\]

where \( s \) represents the arc \( A_1OA_2 \) (Fig. 3) and \( ds \) refers to the differential arc length of \( [1 + (y')^2]^{1/2}dx \) based on
differential calculus. Using this definition of $ds$ in Eq. (8) combined with the substitution of Eq. (7) (i.e., this equation represents the vertical force equilibrium along b) results in

$$L = \frac{2\gamma}{p_0 + \gamma h} \int_0^h y(x) dx + 2\int_0^h \left[ 1 + (y')^2 \right]^{1/2} dx$$

(9)

For a prescribed $L$, the simultaneous solution of Eqs. (3), (5), and (9) yields the relationship between $T$, $p_0$, or $h$ on the one hand and $y(x)$ on the other hand, which is the explicit form of Eq. (4). This solution is complete if one of the design parameters ($T$, $h$, or $p_0$) is specified. The numerical process involved in this solution is rather tedious and requires a trial-and-error procedure. Several computational schemes are available in the literatures (e.g., Liu, 1981; Carroll, 1994; Kazimierowicz, 1994).

A computer program named “Design Flat Geosynthetic Tube” (DFGT) was independently developed by the authors of this paper as a design tool that can allow users to specify various safety factors related to geosynthetic strength. The calculated results are related to geosynthetic strength.

Finally, the axial tensile force per unit length $T_{axial}$ in the geosynthetic shell encapsulating the slurry needs to be assessed. Fig. 3 presents the definition of this force. The force $P$ acting on a vertical plane signifies the end of a tube resulting from pressurized slurry. It is given by

$$P = 2 \int_0^h (p_0 + \gamma h) y(x) dx$$

(10)

The force $P$ is carried by the tube in the z-direction (i.e., axial direction). Thus, the force $T_{axial}$ per unit length is represented by $P$ divided by the circumference $L$ of the tube. Specifically, it is expressed as

$$T_{axial} = \frac{(2/L) \int_0^h (p_0 + \gamma h) y(x) dx}{1}$$

(11)

Upon determining the tube geometry through the solution of Eq. (3), the value of $T_{axial}$ is then computed by solving Eq. (11).

![Fig. 3. Axial tensile force in a geosynthetic tube.](image)

Typically, the circumferential force $T$ is larger than $T_{axial}$. Hence, if a geosynthetic tube with isotropic strength is considered, the value of $T_{axial}$ is unnecessary in the design process.

2. Example of designing a fully inflated tube

An instructive example of designing a fully inflated tube is developed with the DFGT program. The following factors are observed: the circumference of the tube is set to $L = 9$ m, the unit weight of the slurry is $\gamma = 12$ kN/m$^3$, and the strength of the geosynthetic tube is 24 kN/m. Setting $p_0$ are 0, 5, 20, and 100 kPa to calculate $T$, $h$, $b$, and $B$, respectively (Fig. 2). The results are presented in Table 1 and Fig. 4. This approach is typically used to design sausage-type tubes with large roundness.

<table>
<thead>
<tr>
<th>Pump Pressure $P_0$ (kPa)</th>
<th>Tube Height $h$ (m)</th>
<th>Bottom Width $b$ (m)</th>
<th>Section Width $B$ (m)</th>
<th>Tension Stress $T$ (kN/m)</th>
<th>Section Area $A$ (m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.9</td>
<td>3.6</td>
<td>4.0</td>
<td>2.5</td>
<td>3.3</td>
</tr>
<tr>
<td>5</td>
<td>1.8</td>
<td>2.4</td>
<td>3.6</td>
<td>14.2</td>
<td>5.4</td>
</tr>
<tr>
<td>20</td>
<td>2.3</td>
<td>1.6</td>
<td>3.2</td>
<td>38.2</td>
<td>6.1</td>
</tr>
<tr>
<td>100</td>
<td>2.7</td>
<td>0.6</td>
<td>3.0</td>
<td>154.8</td>
<td>6.4</td>
</tr>
</tbody>
</table>
The calculated results show that the tube must be filled to a certain height (the so-called perfect height) to form a perfect shape when the pumping pressure is equal to zero. In this example, the minimum height (the perfect height, denoted as \( h_p \)) is 0.9 m. When the designed height is less than the perfect height, the existing procedure cannot work. In practice, the designed height for constructing a dike is around 0.5 m. Hence, the existing design method must be improved to solve practical problems.

3. Relationship between the parameters of a fully inflated tube

Using the DFGT program, the geometry and tensile force of a fully inflated geotube can be established. This calculation reflects the ultimate state, and the tube cannot expand under this pressure. However, the shape of the tube changes with an increase in pressure. The relationship among the normalized pumping pressure, normalized height, normalized area, normalized tensile force, and normalized width can be calculated using different unit weights \( \gamma \) and perimeters \( L \), such as \( \gamma = 12kN/m^3 \), \( L = 5m \); \( \gamma = 12kN/m^3 \), \( L = 10m \); \( \gamma = 12kN/m^3 \), \( L = 20m \); \( \gamma = 10kN/m^3 \), \( L = 10m \); \( \gamma = 14kN/m^3 \), \( L = 20m \).

Fig. 5 shows the pumping pressures \( p_0(\gamma L) \) versus the height of the cross-sectional \( H/L \) curve. With the increase in pumping pressure, the normalized height increases rapidly under low pressure and increases slowly under high pressure. When the normalized pressure is greater than 0.5, the curve tends to carry a constant value. A similar relationship between normalized pumping pressure \( p_0(\gamma L) \) and normalized area \( A/L^2 \) is shown in Fig. 6. In practical engineering, the normalized area can be considered as the amount of sand. Hence, it can serve as a design reference for practical projects.

The relationships among the normalized pumping pressure and normalized width of the cross section \( B/L \), normalized contact width with ground \( b/L \), and normalized tensile force \( T/(\gamma L^2) \) are shown in Figs. 7 to 9. Figs. 7 and 8 show that with the increase in pumping pressure, the normalized width of the cross section and the normalized contact width with ground decrease quickly at first, decrease slowly later, and then maintain constant values. Thus, the geometry parameters of the geosynthetic tubes such as \( H/L \), \( A/L^2 \), \( B/L \), and \( b/L \) change quickly at first, change slowly later, and finally maintain constant values. The cut-off point is that when the normalized pumping pressure is approximately 0.5.

As shown in Fig. 9, the normalized tensile force \( T/(\gamma L^2) \) increases linearly with the normalized pumping pressure.

For fully inflated tubes, the geometry and tensile force are fixed under one pressure with corresponding geometry parameters. The relationship among the normalized width of the cross section \( B/L \), normalized contact width with ground \( b/L \), normalized tensile force \( T/(\gamma L^2) \), normalized area \( A/L^2 \), and normalized height of the cross section \( H/L \) are shown in Fig. 10 to 13.
Fig. 7. Relationship between $p_0/\gamma L$ and $B/L$.

Fig. 8. Relationship between $p_0/\gamma L$ and $b/L$.

Fig. 9. Relationship between $p_0/\gamma L$ and $T/(\gamma L^2)$.

Fig. 10. Relationship between $H/L$ and $B/L$.

Fig. 11. Relationship between $H/L$ and $b/L$.

Fig. 12. Relationship between $H/L$ and $T/(\gamma L^2)$.

Fig. 13. Relationship between $H/L$ and $A/L^2$.

Figs. 10 and 11 show the decrease in the quadratic functions of the normalized width of the cross section and normalized contact width with ground.

Similarly, Fig. 12 shows that the normalized tensile force increases slowly then quickly. For fully inflated tubes, the geometry and tensile force can be calculated according to their relationship with the normalized height. This approach ensures that the force does not exceed the strength of the material. Fig. 13 shows that the normalized area increases with pressure.

To verify the accuracy of the proposed method, the predictions made using the proposed method are compared
with the solutions given by other analytical methods. The comparison involves the design charts developed by Cantré (2002), as shown in Figs. 14(a) and 14(b), for the $p_0/(\gamma L)$ vs $H/L$ and $H/L$ vs $T/(\gamma L^2)$ relationships, respectively. The design charts by Cantré are based on a non-dimensional method proposed by Plaut and Suherman (1998). With the use of the proposed method, the normalized pumping pressure versus height of the geosynthetic tube can be calculated using Fig. 7 and plotted in Fig. 14(a). Using Fig. 12, a relationship between $T/(\gamma L^2)$ and $H/L$ is plotted in Fig. 14(b). Fig. 14 shows the good agreement between the proposed solution and that of Cantré.

![Graph](image1.png)

(a) Comparison of $p_0/(\gamma L)$ vs $H/L$ curves

![Graph](image2.png)

(b) Comparison of $H/L$ vs $T/(\gamma L^2)$ curves

Fig. 14. Comparisons of the results obtained with the proposed method and those of Cantré (2002).

### III. DESIGN OF A FLAT GEOSYNTHETIC TUBE WITH A HEIGHT THAT IS LESS THAN THE PERFECT HEIGHT

#### 1. Procedure for designing a partially inflated tube

Partially inflated geotubes are not inflated fully under one pressure, and their height is less than the perfect height. In other words, the state of partially inflated geotubes is one moment in the progress of pumping. To determine the shape of the tube with a height that is less than the perfect height, a true situation is considered: when a small amount of slurry is poured into the tube, the tube is inflated to a certain height $h$, which is less than the perfect height (Fig. 15). A comparison of Figs. 2 and 15 shows that on each side of the tube, the loading condition is $p(x) = p_0 + \gamma(x)$ (note: $p_0 = 0$). Moreover, the geometrical boundary condition at point O must be horizontal to ensure a smooth transition from one-half of the tube of the symmetrical problem to the other half, that is, $1/y'(0) = 0$. Therefore, this part of the tube can be determined using existing theory. These curves at the two sides of the tube can be joined to form a perfectly shaped tube. The calculation results show that with $p_0 = 0$, the total length of the circumference of the formed tube is less than that of the designed tube if $h$ is less than the perfect height. Fig. 16 shows the real figure of this type of tube, in which a horizontal line segment joins the curves at the two sides of the tube. Obviously, the line segment meets the requirements of the geometrical boundary condition [Eq. (4)] and the vertical force equilibrium along the flat base length [Eq. (6)].

At this stage, the pipe pressure can only act to expedite the filling of materials. However, the pipe pressure cannot be sustained inside the tube because no component of tension force exists at the line segment to balance the normal pressure.

![Diagram](image3.png)

Fig. 15. Loading and boundary conditions of a partially inflated tube.

On the basis of the discussion presented above, an
iteration procedure is developed to determine the shape of the partially inflated tube (i.e., the deigned height of the bag denoted as \( h \) is less than the perfect height). The steps are detailed below.

1. Determine the parameters for the design: the unit weight of the slurry \( \gamma \), the desired height \( h \), and the perimeter of the tube \( L \).

2. Assume a fully inflated trial tube with a perimeter of \( L_c \) (\( L_c < L \)).

3. Calculate the height of the trial tube \( h_c \) using the DFGT program with the given \( \gamma \) and assumed \( L_c \) and \( p_0 = 0 \). If \( h_c \neq h \), repeat steps (2) to (3) until \( h_c = h \) is reached. Record \( L_c \) and the tension force \( T \).

4. The length of the line segment denoted as \( L_s \) is one-half of the difference between the perimeter of the designed tube \( L \) and that of the trial tube \( L_c \) (Fig. 15). It is given by

   \[
   L_s = (L - L_c)/2 \tag{12}
   \]

5. Define the shape of the designed tube once the \( L_s \) is found.

This iterative procedure is included in the DFGT program for the design of fully inflated and flat tubes.

2. Example of designing a partially inflated tube

For this example, the design conditions are considered the same as those presented in Section 3.1, that is, \( \gamma = 12 kN/m^3 \), \( L = 9 \)m and \( p_0 = 0 \). The procedure developed in Section 3.2 is used to calculate the results with different tube heights. The results are presented in Fig. 15 and Table 2.

![Fig.16. Shape of a partially inflated geotube.](image)

When \( H = 0.9 \) m, the partially inflated geotube is in the ultimate state. The calculation result is the same as that for a fully inflated tube, thus proving the consistency and uniformity between the two programs. In other words, the two programs can be used in different situations. When the pumping pressure, weight, and perimeter are established, the perfect height is calculated. When the design height is less than the perfect height, the partially inflated geotube program should be used, otherwise the fully inflated program should be used.

3. Relationship between the parameters of a partially inflated tube

The height of partially inflated tubes is an important controlling factor; hence, the relationship among the normalized width of the cross section \( B/L \), normalized contact width with ground \( b/L \), normalized tensile force \( T/(\gamma L^2) \), normalized area \( A/L^2 \), and normalized height of the cross section \( H/L \) must be established. Figs. 18 to 21 show the curves when the pumping pressure is zero. These relationships reflect the process in which the parameters change regularly when the tube is empty and when the tube is full. Figs. 18 to 19 show that the normalized width of the cross section and normalized contact width with ground decrease linearly with the normalized pumping pressure.

Figs. 20 and 21 show that with an increase in the normalized height, the quadratic functions of the normalized tensile force and normalized area increase.
Table 2. Calculation results for a flat tube.

<table>
<thead>
<tr>
<th>Tube Height $H$ (m)</th>
<th>Bottom Width $b$ (m)</th>
<th>Line Segment Width $L_s$ (m)</th>
<th>Section Width $B$ (m)</th>
<th>Tension Stress $T$ (kN/m)</th>
<th>Section Area $A$ (m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>4.2</td>
<td>3.3</td>
<td>4.36</td>
<td>0.29</td>
<td>1.27</td>
</tr>
<tr>
<td>0.5</td>
<td>4.0</td>
<td>2.3</td>
<td>4.26</td>
<td>0.78</td>
<td>2.02</td>
</tr>
<tr>
<td>0.7</td>
<td>3.8</td>
<td>1.2</td>
<td>4.17</td>
<td>1.50</td>
<td>2.67</td>
</tr>
<tr>
<td>0.9</td>
<td>3.6</td>
<td>0</td>
<td>4.04</td>
<td>2.50</td>
<td>3.30</td>
</tr>
</tbody>
</table>

Fig. 18. Relationship between $H/L$ and $B/L$.  

Fig. 19. Relationship between $H/L$ and $b/L$.  

Fig. 20. Relationship between $H/L$ and $T/(\gamma L^2)$.  

Fig. 21. Relationship between $H/L$ and $A/L^2$.  

V. CONCLUSIONS

Using tubes made of sewn geosynthetic sheets to construct dikes for land reclamation is a widely used technique. The iterative procedure presented in this paper can determine the relationships among tube size, pumping pressure, unit weight of the slurry, and tension stress in geosynthetic sheets. The proposed procedure is effective in designing both fully inflated and flat tubes. The validity of the numerical procedure used to solve the resulting equations is verified through a comparison of the results with the numerical and experimental results obtained by other investigators.

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NOTATIONS

$A$ Area of cross section  
$b$ Contact width with subgrade
\[ B \] Width of cross section
\[ H \] Height of cross section
\[ H_c \] Height of the trial tube using DFGT
\[ L \] Perimeter of cross section
\[ L_c \] Perimeter of a fully inflated trial tube cross section
\[ L_s \] Length of the line segment of cross section
\[ L_p \] Width of the load on the formed tube
\[ P_o \] Pumping pressure
\[ P_o' \] Inner pressure
\[ P_L \] Load induced by the weight of the tubes in the upper portion
\[ P \] Radius of the infinitesimal element
\[ T \] Tensile force along the geosynthetic tube per unit length
\[ T_{axial} \] Axial tensile force per unit length
\[ W \] Total weight of filling material
\[ \gamma \] Unit weight of filling slurry
\[ \theta \] Angle between the x-axis and the tangential direction at a point

REFERENCES


